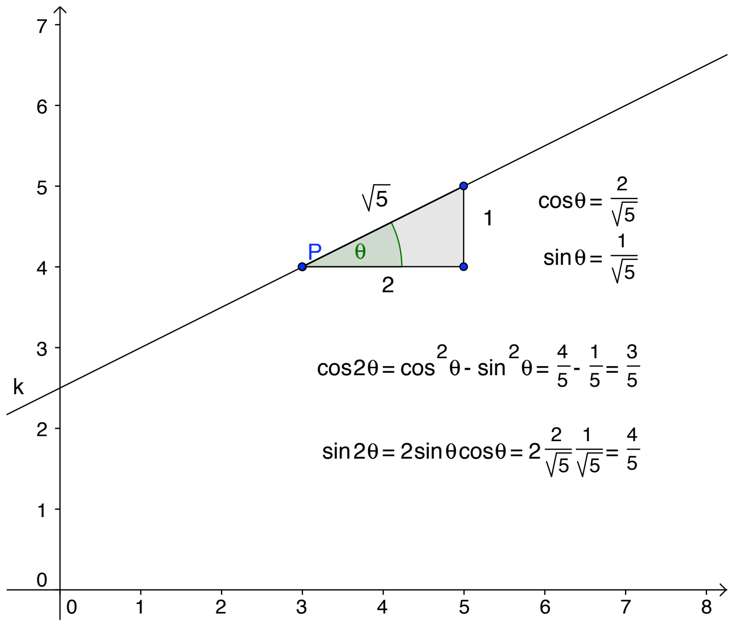
In class we constructed the matrix formula for the reflection across the line $k = \{(3,4) + t(2,1)\}$ – i.e. the line through $P = (3,4)$ with direction indicator $U = (2,1)$. The picture below shows both the line and how to find $\cos\theta$, $\sin\theta$, $\cos 2\theta$ and $\sin 2\theta$, where θ is the angle formed by k and the horizontal.



Using the formula defined in class (together with the trig values found above),

$$\begin{aligned}
\mathcal{M}_k(X) &= F_\theta(X - P) + P \\
&= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix} \left(X - \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right) + \begin{bmatrix} 3 \\ 4 \end{bmatrix} \\
&= \begin{bmatrix} 3/5 & 4/5 \\ 4/5 & -3/5 \end{bmatrix} \left(X - \begin{bmatrix} 3 \\ 4 \end{bmatrix} \right) + \begin{bmatrix} 3 \\ 4 \end{bmatrix}
\end{aligned}$$

If we multiply this out, we have:

$$\begin{aligned}
\mathcal{M}_k(X) &= \begin{bmatrix} 3/5 & 4/5 \\ 4/5 & -3/5 \end{bmatrix} X - \begin{bmatrix} 3/5 & 4/5 \\ 4/5 & -3/5 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix} \\
&= \begin{bmatrix} 3/5 & 4/5 \\ 4/5 & -3/5 \end{bmatrix} X - \begin{bmatrix} 5 \\ 0 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix} \\
&= \begin{bmatrix} 3/5 & 4/5 \\ 4/5 & -3/5 \end{bmatrix} X + \begin{bmatrix} -2 \\ 4 \end{bmatrix}
\end{aligned}$$

In class, with 2:15pm approaching, I quoted $-\begin{bmatrix} 8 \\ 4 \end{bmatrix}$ from memory instead of the correct $+\begin{bmatrix} -2 \\ 4 \end{bmatrix}$. Grrr!

We had two calculations:

$$\mathcal{M}_k\left(\begin{bmatrix}5\\0\end{bmatrix}\right) = \begin{bmatrix}3/5 & 4/5\\4/5 & -3/5\end{bmatrix}\begin{bmatrix}5\\0\end{bmatrix} + \begin{bmatrix}-2\\4\end{bmatrix} = \cdots = \begin{bmatrix}1\\8\end{bmatrix}$$
$$\mathcal{M}_k\left(\begin{bmatrix}0\\5\end{bmatrix}\right) = \begin{bmatrix}3/5 & 4/5\\4/5 & -3/5\end{bmatrix}\begin{bmatrix}0\\5\end{bmatrix} + \begin{bmatrix}-2\\4\end{bmatrix} = \cdots = \begin{bmatrix}2\\1\end{bmatrix}$$

You can check on the picture above that the reflection of $(5,0)$ across k is in fact $(1,8)$, and the reflection of $(0,5)$ is $(2,1)$. One further note: occasionally we might write the vector X in terms of its components x and y, in which case the formula takes the form:

$$\mathcal{M}_k\left(\begin{bmatrix}x\\y\end{bmatrix}\right) = \begin{bmatrix}3/5 & 4/5\\4/5 & -3/5\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} + \begin{bmatrix}-2\\4\end{bmatrix} = \cdots = \begin{bmatrix}\frac{3}{5}x + \frac{4}{5}y - 2\\ \frac{4}{5}x - \frac{3}{5}y + 4\end{bmatrix}$$

Or, writing our vectors out with parentheses instead:

$$\mathcal{M}_k(x,y) = \left(\frac{3}{5}x + \frac{4}{5}y - 2, \frac{4}{5}x - \frac{3}{5}y + 4\right)$$