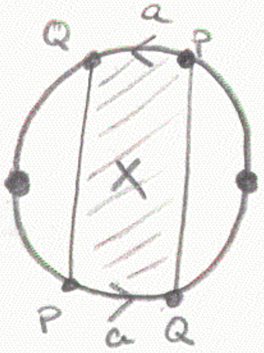
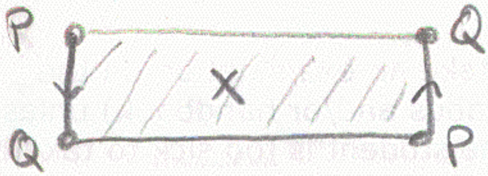
Math 5345 F07 – HW #5

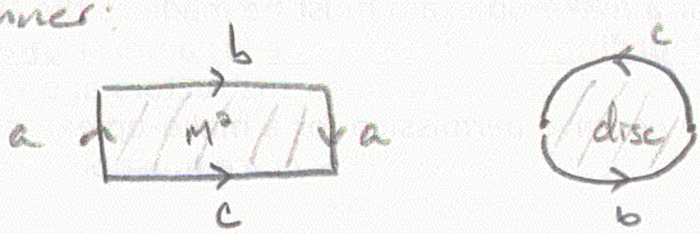
3.1 #3 (a) Show P^2 contains a Möbius band.

Use a std gluing diagram for P^2; the shaded subset X will be homeomorphic to M^2:

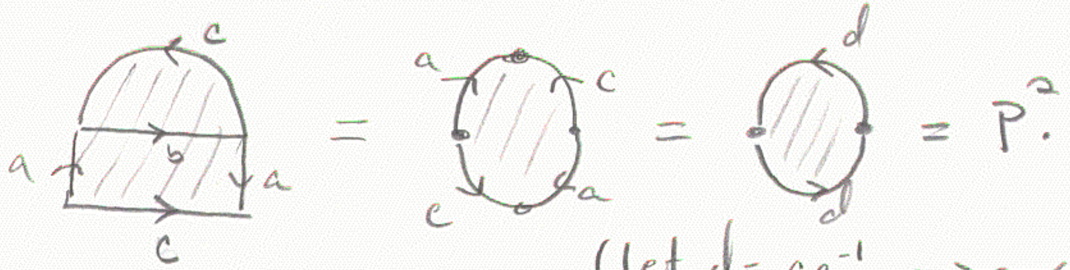




(b) This was an example in class. The key is to label the edges of the disc in a "useful" manner:



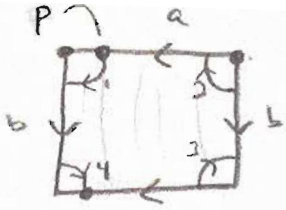
Note the bdies of M^2 and the disc are both b followed by c, so gluing these edges together gives the desired surface. To show the surface is P^2, actually put the b's together:



(let $d = ca^{-1}$; 

)

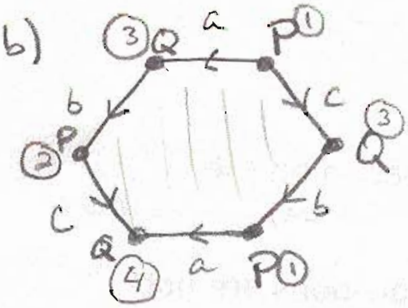
3.1 #4 (a) Start at P and go clockwise:



after gluing edges, the 4 quarter-circles become a disk:

The interior of this disk is an open disk ($\cong \mathbb{R}^2$) containing the corner point of the rectangle.

(b)

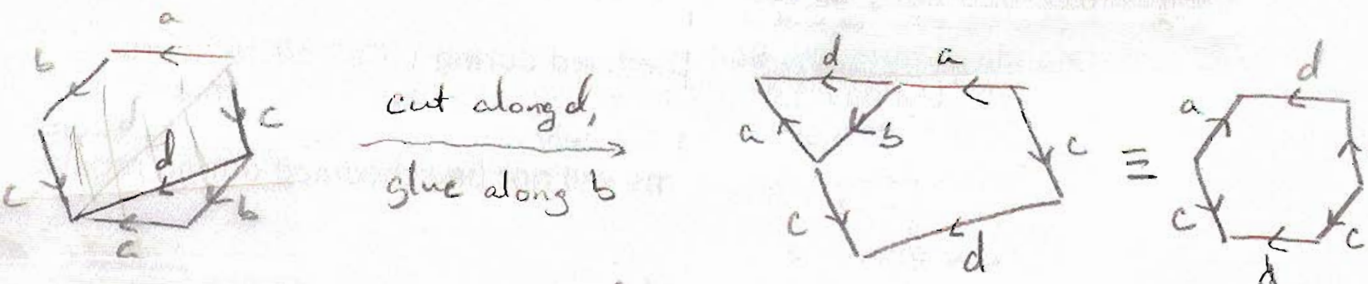


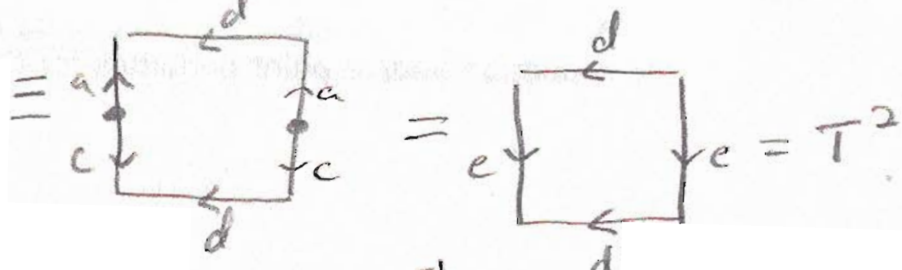
P = ① starting point of a, ② also the starting point of c, which also makes it the ending point of b, and that's it.

Q = ③ starting pt of b, which also makes it ④ the end of a and end of c.

So the 6 vertices are glued into two groups of three.

(c) [Note: I showed you this gluing diagram in class on an overhead, it gives a torus.]

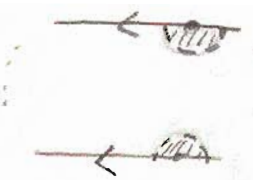




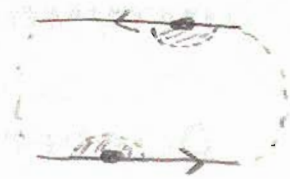
$e = a^{-1}c$,

3.2 #1 (a) Think of the polygon as a subset of $\mathbb{R}^2$. By def$^{\underline{n}}$ if x is an interior pt, then $B_r(x) \subset$ Polygon for some $r>0$.

(b)

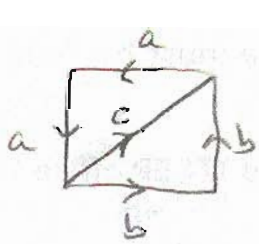


or

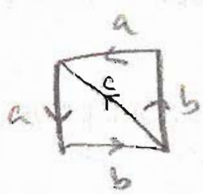


(c) This is like 3.1 #4(a), where you trace around vertices and get quarter circles which assemble into a disc. It may not always be quarters, but you will get a disk. I'm ok if you didn't do a rigorous proof that you eventually get back to where you started, etc

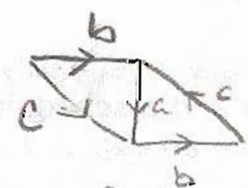
3.2 #7 (a)



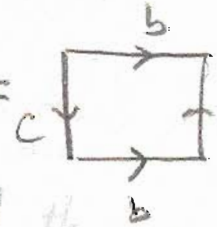
won't work; need to have both a, b on each triangle so you can glue them together along an edge other than c.



$\xrightarrow[\text{glue along } a]{\text{cut along } c}$



$=$



which is a Klein Bottle, and th

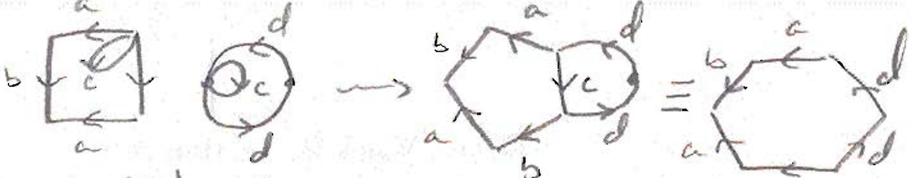
that's what's given as $aba^{-1}b$:

(Can get that from my diagram by relabeling edges, making sure to remember that the original a, b's not the same as the ones at the end.

(b) By example 3.15,

$= P^2 \# P^2$.

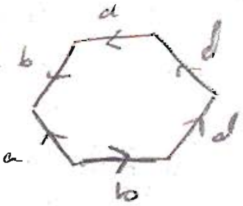
3.2 #8 (a) $T^2 \# P^2$:



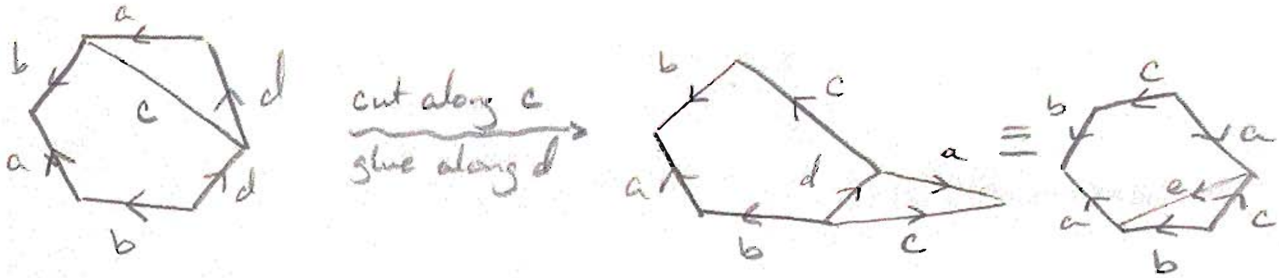
$aba^{-1}b^{-1}$, dd

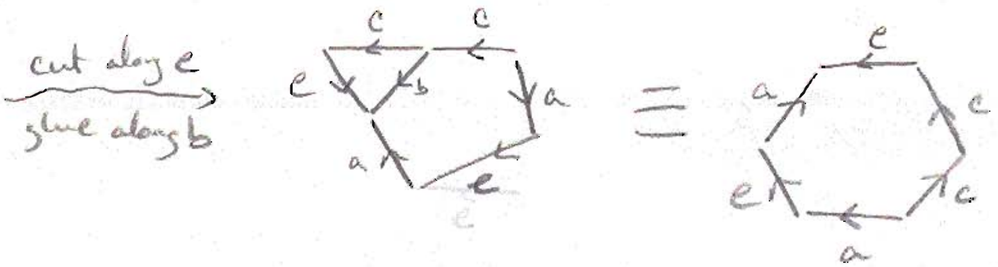
$aba^{-1}b^{-1}dd$

$K^2 \# P^2$: Similar; results in $aba^{-1}\underline{b}dd$:

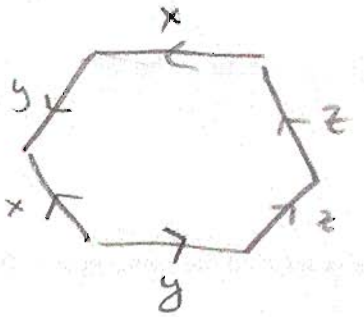


(b) Start with $T^2 \# P^2$





relabeling gives
$\begin{pmatrix} x = e \\ y = a^{-1} \\ z = c \end{pmatrix}$



$(xyx^{-1}yzz)$

which matches the pattern for gluing diagram of $K^2 \# P^2$ in (a).

(c) In (b), $T^2 \# P^2 \equiv K^2 \# P^2$, but we can't "cancel" the P^2's since $T^2 \not\equiv K^2$.