

1.(i) D can be described by $-y_0 \leq y \leq y_0$, $y^2 \leq x \leq \sqrt{1-y^2}$.

1.(ii) Start with $x = y^2 = \sqrt{1-y^2}$, or $y^4 = 1 - y^2$. Letting $u = y^2$, this is the quadratic equation $u^2 + u - 1$, which has solutions at $u = \frac{-1 \pm \sqrt{5}}{2}$. Solving for y we have $y = \pm \sqrt{\frac{-1 \pm \sqrt{5}}{2}}$. We want the positive real solution, which is $\sqrt{\frac{-1 + \sqrt{5}}{2}}$.

1.(iii)

$$\iint_D xy^4 dA = \int_{-y_0}^{y_0} \int_{y^2}^{\sqrt{1-y^2}} xy^4 dA = \int_{-y_0}^{y_0} \left[\frac{x^2 y^4}{2} \right]_{y^2}^{x=\sqrt{1-y^2}} dA = \int_{-y_0}^{y_0} \frac{1}{2} (y^4 - y^6 - y^8) dA = \frac{y_0^5}{5} - \frac{y_0^7}{7} - \frac{y_0^9}{9}$$

2.(i) Set $u = xy$ and $v = x/y$, so $2 \leq u \leq 4$ and $\frac{1}{2} \leq v \leq 3$. Note that $uv = x^2$; hence $x = \sqrt{uv}$. Similarly, $y = \sqrt{u/v}$. Hence our transformation is

$$\begin{aligned} x &= \sqrt{uv} & 2 \leq u \leq 4 \\ y &= \sqrt{u/v} & \frac{1}{2} \leq v \leq 3 \end{aligned}$$

2.(ii) The correct absolute value of the Jacobian for this change of variables is $\frac{1}{2v}$. Thus our integral becomes

$$\iint_D x^2 y^2 dA = \int_2^4 \int_{1/2}^3 (uv) \frac{u}{v} \frac{1}{2v} dv du = \int_2^4 \int_{1/2}^3 \frac{u^2}{2v} dv du = \dots = \frac{26 \ln 6}{3}$$

3. By symmetry, $\iint_D f dA = 2 \iint_{D_1} f dA$. Switching to polar coordinates,

$$2 \iint_{D_1} [2(x^2 + y^2) + 1] dx dy = 2 \int_{\pi/6}^{\pi/2} \int_0^4 2r^3 + r dr d\theta = \dots = \frac{272\pi}{3}$$

4. This problem is impossible unless you change the order of integration!

$$\int_0^2 \int_{y^2}^4 y e^{x^2} dx dy = \int_0^4 \int_0^{\sqrt{x}} y e^{x^2} dy dx = \int_0^4 \frac{1}{2} x e^{x^2} dx = \dots = \frac{e^{16} - 1}{4}$$

5. $\int_{-\pi}^{\pi} \sin(y+c) dy = 0$ for any constant c . Thus if you set this integral up as $dy dx$, the inner integral evaluates to 0 and the entire answer is zero.

6.(i) The first grid curve is a circle of radius $R/2$ centered at $(1/2, 1/2, M/2)$ in the plane $z = M/2$. The second curve is similar; replace $1/2$ with 1.

6.(ii) S is the slanted cone on the left; ask us for reasons!