Solutions to Exam 2

- 1.(a) Ask us if you're not sure what the pictures look like.
- (b) $\int_0^2 \int_0^{3-3z/2} \int_0^{2-2y/3-z} f(x, y, z) dx dy dz$ (c) $\int_0^2 \int_3^{3-3z/2} 2 - \frac{2}{3}y - z dy dz$; other answers possible.

2. (a) Again, ask us about the picture. For (b),

$$V(S) = \iiint_{S} 1 \, dV = \int_{0}^{2\pi} \int_{\pi/4}^{\pi/3} \int_{0}^{4} \rho^{2} \sin \phi \, d\rho \, d\phi \, d\theta = 2\pi \frac{64}{3} \int_{\pi/4}^{\pi/3} \sin \phi \, d\phi = \dots = \frac{64\pi}{3} \left(\sqrt{2} - 1\right)$$

3. (i) This is the half cone $z = \sqrt{x^2 + y^2}$ for $0 \le z \le 1$. (ii) $\vec{n} = \vec{r_u} \times \vec{r_v} = \langle -u \cos v, -u \sin v, u \rangle$, and $\vec{n}(1/2, \pi/2) = \langle 0, -1/2, 1/2 \rangle$. (iii) $A(S) = \int_0^1 \int_0^{2\pi} |\vec{r_u} \times \vec{r_v}| \, dv \, du = \cdots = 2\pi\sqrt{2} \int_0^1 u \, du = \pi\sqrt{2}$.

4. (a) One possibility is a clockwise half circle.

(b) One possibility is a straight line segment, up the *y*-axis.

(c) The line integral over the segment on the y-axis is zero; the other part is positive; hence the entire thing is positive.

(d) One possibility is a counter-clockwise circle around the origin.

5. (a) Ask us about the picture.

(b) $\int_C \vec{F} \cdot d\vec{r} = \int_{3\pi/4}^{7\pi/4} \langle -\sin^2 t, \cos^2 t \rangle \cdot \langle \sin t, -\cos t \rangle dt = \cdots = -\int_{3\pi/4}^{7\pi/4} \sin^3 t + \cos^3 t \, dt$. This eventually boils down to $-5\sqrt{2}/3$.

6. (a) Ask us about the picture

(b) Let $x = u^3$, $y = v^3$, and $z = w^3$. Then the area expansion factor is simply $27u^2v^2w^2$. In *uvw*-space, the domain is the tetrahedron in the first octant, cut off by the plane u + v + w = 8. The integral is

$$V(S) = \iiint_{S} dV = \int_{0}^{S} \int_{0}^{s-u} \int_{0}^{s-u-v} 27u^{2}v^{2}w^{2} dw dv du$$

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