1. The first row, from left to right, is **a** and **b**. The second row, from left to right, is **c** and **d**. To distinguish between the graphs, you could look at the behavior of the z-coordinate: does a certain graph exhibit linear growth? Polynomial growth? (Note that t^3 does not grow exponentially! Exponential growth would be e^t .) Also note that **c** has considerably more revolutions than the others, and **d** lies on the surface of a cone, because $\sqrt{x(t)^2 + y(t)^2} = z(t)^2$.

2. (i) $\mathbf{r}(t) = (2\cos t)\mathbf{i} + (2\sin t)\mathbf{j} + t\mathbf{k}$

(ii)
$$\mathbf{r}(t) = (4\sqrt{3}\cos t)\mathbf{i} + (4\sqrt{3}\sin t)\mathbf{j} + 4\mathbf{k}$$

- 3. (i) $s(t) = \int_0^t |\mathbf{r}'(u)| \, du = \int_0^t \sqrt{(-a \sin u)^2 + (b \cos u)^2 + 1} \, du.$
 - (ii) With $a = b = \sqrt{3}$, this becomes $s(t) = \int_0^t \sqrt{3+1} \, du = \int_0^t 2 \, du = 2t$, so $s(6\pi) = 12\pi$.
 - (iii) $\kappa(t) = \sqrt{3}/4$. Since $\kappa(t)$ does not depend on t, the curve has constant curvature.

4.
$$x^2/9 + y^2/16 + z^2/1 = 1$$
.



5. (i) The key here was to draw tangent *vectors*, not tangent lines. Because they are unit tangent vectors, you should have drawn them with the same length. Also, because the particle is moving at a constant speed, the acceleration vectors are always perpendicular to the velocity (i.e. tangent) vectors. At point A the turn is much sharper, so the acceleration vector should be longer than at point C.

(ii) The osculating circle for a curve at $\mathbf{r}(t)$ is the circle which "best fits" the curve at that point. It has radius $1/\kappa(t)$.

(iii) Intuitively, the curve is flattest (or "least curvy") at C, followed by B, and then A. You could also do this problem by considering the radii of the osculating circles at those points.

Jonathan Rogness <rogness@math.umn.edu>