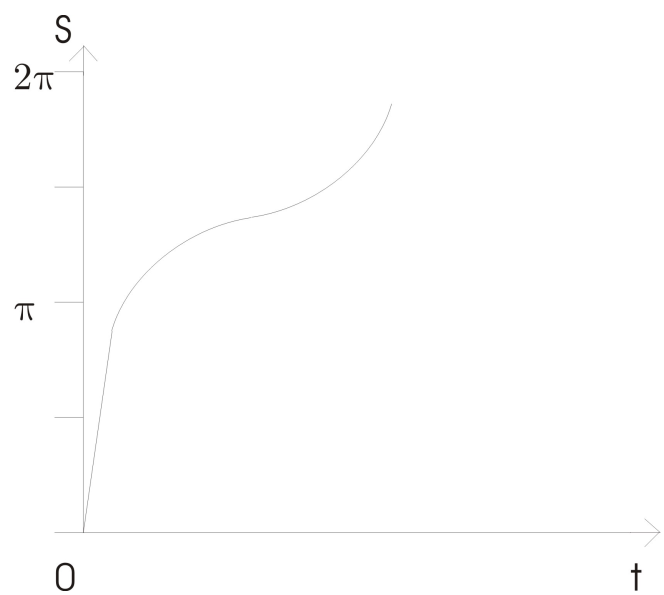
This exam covers sections 9.6, 10.5, 11.1, and 11.2. It also covers the material from sections 10.3 and 10.4 which weren't on the previous exam. This includes TNB frames, osculating planes and components of acceleration. In class we mentioned applicable problems from the chapter reviews in your textbook: p702 27-36; p743 10, 11, 20, 21; and p833 27-36. Here are some other suggested problems.

1. Sketch and describe the level sets (curves) of $f(x, y) = (x - y)^2$ in the xy-plane.

2. Describe the level sets (surfaces) of the function $g(x, y, z) = z - (x - y)^2$. Sketch the level set $g(x, y, z) = 0$.

3. Sketch the "rollercoaster" curve $\mathbf{r}(t) = \langle \cos t, \sin t, \sin(2t)\rangle$, $0 \leq t \leq 2\pi$. At which points on the curve is the acceleration greatest, and in which direction is it pointing at those places? (Note that your tummy may well feel as if it is moving in the opposite direction to the acceleration.) Can you find the osculating plane when $t = 0$ or $t = \pi/2$?

4. A particle moves in time along the path $\langle \cos(s(t)), \sin(s(t))\rangle$ for values of s and t as shown on the graph below:



Sketch the velocity and acceleration vectors at the points $(0, 1)$, $(-1, 0)$, and $(0, -1)$.

5. Find the limits (if they exist) of the following functions as $(x, y) \to (0, 0)$.

$$f(x,y) = \begin{cases} (x^2 + y^2)\cos(x + 3y^2), & (x,y) \neq (0,0) \\ 1, & (x,y) = (0,0) \end{cases} \qquad g(x,y) = \begin{cases} e^{(-1/x^2)}, & x \neq 0 \\ 10, & x = 0 \end{cases}$$

6. Consider the curve parametrized by

$$\mathbf{r}(t) = \begin{cases} \langle t-2, 1, 0\rangle, & 0 \leq t \leq 2 \\ \langle \sin(t-2), \cos(t-2), t-2\rangle, & 2 \leq t \leq 2+\pi/2 \\ \langle 1, t-2+\pi/2, 1\rangle, & 2+\pi/2 \leq t \end{cases}$$

Where is the acceleration defined in this interval? Where is the acceleration in the direction of motion? Where is it perpendicular to the direction of motion? Sketch the velocity and acceleration vectors at $t = 2 + \pi/4$, placing their tails at the end of the position vector.

Is the acceleration always horizontal when it is defined? Could you change your answer to that question by reparametrizing the curve?

7. Sketch and describe the surfaces in $\mathbf{R}^3$ determined by the following equations.

$$\frac{x^2}{4} - \frac{y^2}{16} + \frac{z^2}{9} = 1$$

$$x^2 + \frac{y^2}{9} - \frac{z^2}{16} = 0$$

$$y^2 + 2 = z$$

8. Let $\mathbf{r}(t) = \left\langle t, t^2, 0\right\rangle$, $0 \leq t \leq 2$. Write down matrices $A(\phi)$, $B(\phi)$, and $C(\phi)$ which represent rotation around the x-, y-, and z-axes by an angle of ϕ. Find the parametric equations that result when you multiply these matrices by $\mathbf{r}(t)$, written as a column vector on the right. Describe the resulting parametric surfaces. (Assume $0 \leq \phi \leq 2\pi$).