

1. Assuming the first row of pictures is labeled I and II, the second is III and IV, and the third is V and VI:

$z = \frac{2}{15}e^{-x-y}$: III. This is the only graph which increases rapidly in Quadrant III (where $-x - y$ is positive, so e^{-x-y} is large) and very small in Quadrant I (where $-x - y < 0$).

$z = \frac{7}{1+(x+y)^2}$: II. This is the only surface which has a height of 7 above every point on the line $y = -x$.

$z = \frac{6 \sin(2(x^2+y^2))}{x^2+y^2}$: V. The radial symmetry narrows it down to graph V or VI. The cross sections $x = 0$ or $y = 0$ in V are consistent with the function $f(u) = \frac{6 \sin(2u)}{u}$.

$z = 5 \sin(3x) + 5 \cos(3y)$: IV. The cross sections $y = k$ are consistent with the graph of $f(u) = 5 \sin(3x) + C$, where C is a constant. Similar arguments hold for the cross sections $x = k$.

2. The limit of $\frac{x^3 y}{x^6 + y^2}$ does not exist as $(x, y) \rightarrow (0, 0)$. If you approach along the curve $y = x^3$, the limit is $1/2$. If you approach along $y = -x^3$, the limit is $-1/2$.

Conversely, the limit of $\frac{x^3}{x^2 + y^2 + z^2}$ as $(x, y, z) \rightarrow (0, 0, 0)$ *does* exist. If we switch to spherical coordinates, the limit becomes

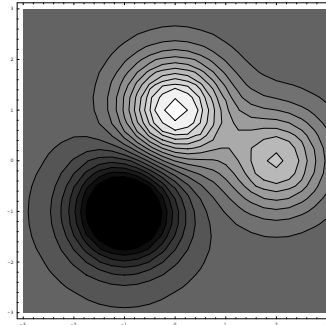
$$\begin{aligned} \lim_{(x,y,z) \rightarrow (0,0,0)} \frac{x^3}{x^2 + y^2 + z^2} &= \lim_{\rho \rightarrow 0} \frac{\rho^3 \cos^3 \theta \sin^3 \phi}{\rho^2} \\ &= \lim_{\rho \rightarrow 0} \rho \cos^3 \theta \sin^3 \phi = 0 \end{aligned}$$

3. In order,

- (i) A particle traveling along the curve will approach the origin as $t \rightarrow 0^-$. At $t = 0$ the particle will be at the origin, and as t increases further, the particle will “turn around” and retrace its path.
- (ii) $\mathbf{r}'(t) = \langle t, \sqrt{2}t^3, t^5 \rangle$. As $t \rightarrow 0$, this approaches $\langle 0, 0, 0 \rangle$. As $t \rightarrow \infty$, this diverges to infinity.
- (iii) $\mathbf{T}(t) = \mathbf{r}'(t)/|\mathbf{r}'(t)| = \langle t, \sqrt{2}t^3, t^5 \rangle / (t + t^5)$. The limit of $\mathbf{T}(t)$ as $t \rightarrow 0$ does not exist because the limit is $\langle 1, 0, 0 \rangle$ as $t \rightarrow 0^+$, and $\langle -1, 0, 0 \rangle$ as $t \rightarrow 0^-$. If $t \rightarrow \infty$, then the limit converges to $\langle 0, 0, 1 \rangle$.
- (iv) $\mathbf{T}(1) = \langle 1/2, 1/\sqrt{2}, 1/2 \rangle$. To find $\mathbf{N}(1)$, we begin by calculating $\mathbf{T}'(t)$ and $\mathbf{T}'(1)$. This is a bit messy, but it's $\mathbf{T}'(1) = \langle -1, 0, 1 \rangle$. Now $\mathbf{N}(1) = \mathbf{T}'(1)/|\mathbf{T}'(1)|$, which is $\langle -1/\sqrt{2}, 0, 1/\sqrt{2} \rangle$.
- (v) One possible equation of the osculating plane is

$$\begin{aligned} \mathbf{p}(u, v) &= \mathbf{r}(1) + u\mathbf{T}(1) + v\mathbf{N}(1) \\ &= \langle 1/2, 1/2\sqrt{2}, 1/6 \rangle + u\langle 1/2, 1/\sqrt{2}, 1/2 \rangle + v\langle -1/\sqrt{2}, 0, 1/\sqrt{2} \rangle \end{aligned}$$

4. Here's a computer generated contour graph of the surface. Lighter shades indicate greater values of z .



5. Here are pictures of the grid curves and the surface itself. (You would have to do some more labeling, such as which grid curve is which.)

