MATH 3283W. Sequences, Series, and Foundations: Writing Intensive. Spring 2009
Drill Problems to Midterm Exam #2, March 16.

The Exam will consist of 5 problems. No books, notes, and calculators will be allowed. The exam is mainly based on Sec. 1–5, plus Taylor formula in Sec. 9, in the textbook. If material from later sections will be required, then the corresponding formulations with some comment will be provided together with the assignment. The problems in those sections are recommended for preparation. In addition, you can try the problems in the following list.

#1. Show that the following series convergent, and find its sum

$$\textbf{(a)}\ \sum_{n=1}^{\infty}\left(\sqrt{n+2}-2\sqrt{n+1}+\sqrt{n}\right); \qquad \textbf{(b)}\ \sum_{n=1}^{\infty}\frac{n}{(n+1)(n+2)(n+3)}; \qquad \textbf{(c)}\ \sum_{n=1}^{\infty}\frac{n^2}{n!};$$

#2. Determine whether or not each of the series is convergent or divergent:

$$\textbf{(a)}\ \sum_{n=1}^{\infty}\frac{n^{n+\frac{1}{n}}}{\left(n+\frac{1}{n}\right)^n}; \qquad \textbf{(b)}\ \sum_{n=1}^{\infty}\left(\frac{n-1}{n+1}\right)^{n(n-1)}; \qquad \textbf{(c)}\ \sum_{n=1}^{\infty}\frac{1}{n^{1+\frac{1}{n}}}; \qquad \textbf{(d)}\ \sum_{n=1}^{\infty}\frac{1}{\ln(n!)};$$

$$\textbf{(e)}\ \sum_{n=2}^{\infty}\frac{1}{(\ln n)^{\ln(\ln n)}}; \qquad \textbf{(f)}\ \sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}}-\sqrt{\ln\left(1+\frac{1}{n}\right)}\right); \qquad \textbf{(g)}\ \sum_{n=1}^{\infty}\left(n^{\frac{1}{n^2+1}}-1\right).$$

#3. Find the radius of convergence of the series

$$\sum_{n=0}^{\infty}\left(n+\frac{1}{n}\right)^{n^2}x^n.$$

#4. Find the power series (together with radius of convergence) for the functions

$$\textbf{(a)}\ \frac{x}{(1-x)(1-x^2)}\ \text{ in powers of } x; \qquad \textbf{(b)}\ \ln\left(\frac{1}{2+2x+x^2}\right)\ \text{ in powers of } x+1.$$

#5. Find $f^{(1000)}(0)$, where

$$f(x)=\frac{1}{1+x+x^2+x^3}.$$