MATH 3283W. Sequences, Series, and Foundations: Writing Intensive.
Midterm Exam #1. March 5, 2009.

60 points are distributed between 5 problems, 12 points each.
You have 50 minutes to work on these problems.

No books, no calculators. Notes (handwritten or typed) are permitted.

#1. Rewrite the sentence $P\&Q = P \vee Q$ in a simpler form.

#2. Let $f(x)$ be a continuous function on $[0,1]$, such that

$$0 < a \le f(x) \le b < 1 \qquad \text{for all} \quad x \in [0,1].$$

Here a and b are given constants. Show that there is a point $x \in (0,1)$ such that $f(x) = x$.

#3. Using the *binomial formula*

$$(1+x)^n = \sum_{k=0}^{n} \binom{n}{k} x^k, \quad \text{where} \quad \binom{n}{k} = \frac{n!}{k!(n-k)!},$$

$(0! = 1! = 1,\ k! = 1 \cdot 2 \cdot \dots \cdot k$ for $k \ge 2)$, simplify the expression

$$S_n = \binom{n}{1} + 2 \cdot \binom{n}{2} + 3 \cdot \binom{n}{3} + \dots + n \cdot \binom{n}{n}.$$

#4. Find the limits

$$(a)\ \text{(5 points)} \ \lim_{n\to\infty} \left(\sqrt[4]{n^4 + n^3} - n \right), \quad (b)\ \text{(7 points)} \ \lim_{n\to\infty} \frac{2^n n!}{n^n}.$$

#5. Consider the sequence

$$s_1 = 0,\ s_2 = 1,\ \text{ and } \ s_{n+2} = \frac{1}{3}(s_n + 2s_{n+1}) \ \text{ for } \ n = 1, 2, \dots.$$

Determine whether this sequence is convergent or divergent. If convergent, find its limit.