

MATH 4512. Differential Equations with Applications.  
Midterm Exam #2. April 15, 2009.

60 points are distributed between 5 problems, 12 points each.  
You have 50 minutes to work on these problems.  
No books, notes, and calculators.

1. Consider a linear homogeneous equation  $Ly = ay'' + by' + cy = 0$  with constant coefficients  $a, b, c$ , which are **strictly positive**. Show that for an arbitrary solution to this equation we have

$$y(t) \rightarrow 0 \quad \text{as } t \rightarrow +\infty.$$

2. Find the general solution of the equation

$$Ly = t^2 y'' - ty' + y = 4t \ln t \quad \text{for } t > 0,$$

given that  $y_1(t) = t$  and  $y_2(t) = t \ln t$  are independent solutions to the homogeneous equation  $Lu = 0$ .

3. Find the general solution of the equation

$$y^{(5)} + 8y''' + 16y' = 0.$$

4. Find the coefficients  $a_n$  in the power series

$$\frac{x}{x^2 - 5x + 6} = \sum_{n=0}^{\infty} a_n x^n,$$

and determine its radius of convergence.

*Hint:*

$$\frac{1}{a - x} = \sum_{n=0}^{\infty} \frac{x^n}{a^{n+1}} \quad \text{for } |x| < |a|.$$

5. Find a series solution in powers of  $x$  of the equation

$$(1 - x^2)y'' - xy' + 16y = 0,$$

which satisfies the initial conditions  $y(0) = 1$ ,  $y'(0) = 0$ .