Quiz 2 on PreCalculus II (Math 1151)

Mark your Recitation Session Number: 015 023 025

Name:_____ Student ID:_____ Score:_____

You must show all your work. Correct answer without any step earns zero point.
You **cannot** use calculators in this quiz.

1. (2 points.) a) Given $\sin\theta = -\frac{2\sqrt{2}}{3}$, $\cos\theta = \frac{1}{3}$, find the exact value of $\cot\theta$ and $\sec\theta$.

Solution:

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\frac{1}{3}}{-\frac{2\sqrt{2}}{3}} = -\frac{1}{2\sqrt{2}} = -\frac{\sqrt{2}}{4},$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{1}{3}} = 3.$$

(4 points.) b) Given $\tan\theta = -\frac{1}{2}$, $\sin\theta > 0$, find the exact value of $\cos\theta$ and $\csc\theta$.

Solution: First we can find $\cot\theta$ by the reciprocal identity:

$$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{-\frac{1}{2}} = -2.$$

Then we want to use one of the Pythagorean identities to get $\csc\theta$. We need to find $\csc^2\theta$ first:

$$\csc^2\theta = 1 + \cot^2\theta = 1 + (-2)^2 = 5.$$

As we take square root, note that $\sin\theta > 0$, the reciprocal identity $\csc\theta = \frac{1}{\sin\theta}$ gives that $\csc\theta$ is also positive, so we have:

$$\csc\theta = \sqrt{5}.$$

Then $\cos\theta$ can be found in terms of $\cot\theta$ and $\csc\theta$ by a variation of a quotient identity:

$$\cos\theta = \frac{\cot\theta}{\csc\theta} = \frac{-2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}.$$

Note: it's true that we define $\tan\theta = \frac{y}{x}$, and here we have $\tan\theta = -\frac{1}{2}$, but under ***NO*** circumstances can we just take for granted that

$$\cos\theta = x = 2,$$

$$\sin\theta = y = -1.$$

Actually $\cos\theta$ can never be greater than 1. Stick to using the identities, (esp. those Pythagorean identities), and don't make this kind of mistakes any more.

2. (4 points.) Determine the amplitude and period of $y = -4\sin(\frac{1}{3}x)$.

Solution: The amplitude is given by

$$|A| = |-4| = 4,$$

and the period is given by

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{1}{3}} = 6\pi.$$