

Quiz 4 on PreCalculus II (Math 1151)

Mark your Recitation Session Number: 015 023 025

Name: _____ Student ID: _____ Score: _____

You must show all your work. Correct answer without any step earns zero point.
You **cannot** use calculators in this quiz.

1. (4 points.) Work is not needed in this problem.

(a) Fill in the blanks:

$$\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3};$$
$$\cos\left[\cos^{-1}\left(\frac{2}{3}\right)\right] = \frac{2}{3}.$$

(b) True or false:

$$\sin[\sin^{-1}(-2)] = -2 \text{ false};$$
$$\tan^{-1}[\tan(2\pi)] = 2\pi \text{ false}.$$

Note: for the first one, -2 is not in the domain of \sin^{-1} , so the left hand side is not well-defined; for the second one, 2π is not in the range of \tan^{-1} , so the right hand side should be 0 instead of 2π .

2. (6 points.) Establish the following identities:

(a)

$$1 - \frac{\cos^2 \theta}{1 - \sin \theta} = -\sin \theta$$

Proof:

$$\begin{aligned} \text{LHS} &= 1 - \frac{1 - \sin^2 \theta}{1 - \sin \theta} \\ &= 1 - (1 + \sin \theta) \\ &= -\sin \theta \\ &= \text{RHS} \end{aligned}$$

(b)

$$\frac{\cos^2 \theta + \cot \theta}{\sin^2 \theta + \tan \theta} = \cot^2 \theta$$

Proof: this is equivalent to prove:

$$\cos^2 \theta + \cot \theta = (\sin^2 \theta + \tan \theta) \cot^2 \theta. \tag{1}$$

$$\begin{aligned} \text{RHS in (1)} &= \sin^2 \theta \cot^2 \theta + \tan \theta \cot^2 \theta \\ &= \sin^2 \theta \frac{\cos^2 \theta}{\sin^2 \theta} + (\tan \theta \cot \theta) \cot \theta \\ &= \cos^2 \theta + \cot \theta \\ &= \text{LHS in (1)} \end{aligned}$$