Quiz 5 on PreCalculus II (Math 1151)

Mark your Recitation Session Number: 015 023 025

Name:______________________ Student ID:______________________ Score:______________________

You must show all your work. Correct answer without any step earns zero point.
You **cannot** use calculators in this quiz.

1. (4 points.) Establish the identity:

$$\frac{\cos(\alpha-\beta)}{\sin\alpha\cos\beta} = \cot\alpha + \tan\beta.$$

Proof:

$$\begin{aligned} \text{LHS} &= \frac{\cos\alpha\cos\beta + \sin\alpha\sin\beta}{\sin\alpha\cos\beta} \\ &= \frac{\cos\alpha\cos\beta}{\sin\alpha\cos\beta} + \frac{\sin\alpha\sin\beta}{\sin\alpha\cos\beta} \\ &= \frac{\cos\alpha}{\sin\alpha} + \frac{\sin\beta}{\cos\beta} \\ &= \cot\alpha + \tan\beta = \text{RHS}. \end{aligned}$$

2. (3 points.) Establish the identity:

$$\tan\frac{\theta}{2} = \csc\theta - \cot\theta.$$

Proof:

$$\begin{aligned} \text{LHS} &= \frac{1-\cos\theta}{\sin\theta} \\ &= \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} \\ &= \csc\theta - \cot\theta = \text{RHS}. \end{aligned}$$

3. (3 points.) Find the exact value of the expression:

$$\cos[2\tan^{-1}(-\frac{3}{4})].$$

Solution: denote $\theta = \tan^{-1}(-\frac{3}{4})$, then we know that this means $\tan\theta = -\frac{3}{4}$, and we are evaluating $\cos 2\theta$, by the double angle formula for cos:

$$\cos 2\theta = 2\cos^2\theta - 1.$$

By one of the Pythagorean identities:

$$\sec^2\theta = \tan^2\theta + 1 = (-\frac{3}{4})^2 + 1 = \frac{25}{16},$$

thus we have: $\cos^2\theta = \frac{1}{\sec^2\theta} = \frac{16}{25}$, therefore $\cos 2\theta = 2\cdot\frac{16}{25} - 1 = \frac{7}{25}$.

Note: this question is reduced to a typical one of evaluating a trigonometric function (cos in this case) given the value of another function (tan in this case). Generally when applying Pythagorean identities, one may always need to determine the sign when taking square roots. In this problem, we may find $\sec\theta = +\frac{5}{4}$ by considering that θ is within the range of $\tan^{-1}$, which is the open interval $(-\frac{\pi}{2}, \frac{\pi}{2})$, i.e., θ is either in the first or in the fourth quandrant. And we know that $\cos\theta$ and $\sec\theta$ are positive for those θ. Then we can find $\cos\theta$ and also $\cos 2\theta$ by using the double angle formula above. However, in this particular problem, we don't have to determine the sign of sec, since all we need is $\cos^2\theta$, which is independent of the sign of $\cos\theta$. But be careful in other cases, pay additional attention of the sign determination.