

## Quiz 8 on PreCalculus II (Math 1151)

Mark your Recitation Session Number: 015 023 025

Name: \_\_\_\_\_ Student ID: \_\_\_\_\_ Score: \_\_\_\_\_

You must show all your work. Correct answer without any step earns zero point.  
You **cannot** use calculators in this quiz.

1. (6 points.) Solve the triangle:

$$\alpha = 75^\circ, b = \sqrt{2}, c = \sqrt{3}.$$

Also, evaluate the area of the triangle above.

**Solution:** use Law of Cosines:

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc \cos \alpha \\ &= 2 + 3 - 2\sqrt{2}\sqrt{3} \cos 75^\circ \\ &= 5 - 2\sqrt{6} \frac{\sqrt{6} - \sqrt{2}}{4} \\ &= 5 - (3 - \sqrt{3}) \\ &= 2 + \sqrt{3} \end{aligned}$$

So we get:

$$a = \sqrt{2 + \sqrt{3}} = \frac{\sqrt{6} + \sqrt{2}}{2}.$$

Use Law of Cosines again:

$$\begin{aligned} \cos \beta &= \frac{a^2 + c^2 - b^2}{2ac} \\ &= \frac{2 + \sqrt{3} + 3 - 2}{2 \frac{\sqrt{6} + \sqrt{2}}{2} \sqrt{3}} \\ &= \frac{3 + \sqrt{3}}{3\sqrt{2} + \sqrt{6}} \\ &= \frac{\sqrt{2}}{2} \end{aligned}$$

So we get:

$$\beta = \cos^{-1} \frac{\sqrt{2}}{2} = 45^\circ,$$

and:

$$\gamma = 180^\circ - \alpha - \beta = 180^\circ - 75^\circ - 45^\circ = 60^\circ.$$

The area of this triangle is:

$$A = \frac{1}{2}bc \sin \alpha = \frac{1}{2}\sqrt{2}\sqrt{3} \frac{\sqrt{6} + \sqrt{2}}{4} = \frac{3 + \sqrt{3}}{4}.$$

2. (4 points.) Evaluate the area of the triangle:

$$a = 4, b = 5, c = 6.$$

**Solution:** use Heron's Formula, first compute:

$$s = \frac{a + b + c}{2} = \frac{15}{2},$$

then we have:

$$A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{\frac{15}{2} \frac{7}{2} \frac{5}{2} \frac{3}{2}} = \sqrt{\frac{1575}{16}} = \frac{15\sqrt{7}}{4}.$$