Quiz 9 on PreCalculus II (Math 1151)

Mark your Recitation Session Number: 015 023 025

Name:______________________ Student ID:______________________ Score:______________________

You must show all your work. Correct answer without any step earns zero point.
You **cannot** use calculators in this quiz.

(6 points.) Given the polynomial $f(x) = 4x^5 - 12x^4 - x + 3$,

- Tell the maximum number of zeros that f may have;
- Use Descartes' Rule of Signs to determine how many positive and how many negative zeros f may have;
- List all the potential rational zeros of f;
- Based on all the information above, find first all the real, and then all the complex zeros of f;
- Use the zeros of f to factor f first over the real, and then over the complex numbers.

Solution:

- The maximum number of zeros is given by the degree of f, i.e., 5;
- The sequence of coefficients of f omitting 0 is: $4, -12, -1, 3$.
  There are 2 changes of signs, thus f may have 2 or 0 positive real zeros.
  Substituting $x = -y$, we get $g(y) = -4y^5 - 12y^4 + y + 3$, and the sequence of its nonzero coefficients is: $-4, -14, 1, 3$, which has only 1 change of signs. So f has exactly 1 negative real zero;
- p could be $\pm 1, \pm 3$, and q could be $\pm 1, \pm 2, \pm 4$, thus all potential rational zeros of f are: $\pm 1, \pm 3, \pm\frac{1}{2}, \pm\frac{3}{2}, \pm\frac{1}{4}, \pm\frac{3}{4}$;
- Checking all potential rational zeros above, one may find out that only 3 is a zero. Thus, we can divide f by the linear factor $x - 3$, to get the depressed polynomial: (Here the process of division is omitted.)

  $$f(x) = (x-3)(4x^4 - 1).$$

  Then the problem is reduced to find zeros of $4x^4 - 1 = 0$ by using some factoring techniques:

  $$4x^4 - 1 = (2x^2 - 1)(2x^2 + 1) = (\sqrt{2}x - 1)(\sqrt{2}x + 1)(\sqrt{2}x - i)(\sqrt{2}x + i).$$

  So all complex zeros of f are: $3, \pm\frac{1}{\sqrt{2}}, \pm\frac{i}{\sqrt{2}}$;
- The factored form is obtained immediately after the previous step:

  $$f(x) = (x-3)(\sqrt{2}x - 1)(\sqrt{2}x + 1)(\sqrt{2}x - i)(\sqrt{2}x + i).$$