

Quiz 10 on PreCalculus II (Math 1151)

Mark your Recitation Session Number: 015 023 025

Name: _____ Student ID: _____ Score: _____

You must show all your work. Correct answer without any step earns zero point.
You **cannot** use calculators in this quiz.

1. (2 points.) The rectangular coordinates of a point is $(-\sqrt{6}, \sqrt{2})$, find its polar coordinates.

Solution:

$$r = \sqrt{x^2 + y^2} = \sqrt{(-\sqrt{6})^2 + (\sqrt{2})^2} = \sqrt{6 + 2} = 2\sqrt{2};$$

$$\tan \theta = \frac{y}{x} = -\frac{\sqrt{2}}{\sqrt{6}} = -\frac{1}{\sqrt{3}},$$

Note that the point is in the second quadrant, for finding θ we need to take the inverse function of tan and add π :

$$\theta = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \pi = -\frac{\pi}{6} + \pi = \frac{5\pi}{6}.$$

2. (4 points.) The letters r and θ represent polar coordinates. Write the following equation using rectangular coordinates (x, y) .

$$r = \frac{3}{1 - 2 \cos \theta}.$$

Solution: first simplify the equation, then replace r and $\cos \theta$ by $\sqrt{x^2 + y^2}$ and x respectively:

$$r - 2r \cos \theta = 3$$

$$\sqrt{x^2 + y^2} - 2x = 3$$

We can claim it's done here, but we can further simplify the equation as well:

$$\sqrt{x^2 + y^2} = 2x + 3$$

$$x^2 + y^2 = 4x^2 + 12x + 9$$

$$3x^2 + 12x - y^2 + 9 = 0$$

$$3(x + 2)^2 - y^2 = 3$$

$$(x + 2)^2 - \frac{y^2}{3} = 1$$

So this is an equation of a hyperbola.

3. (4 points.) Write the expression in the standard form $a + bi$ using De Moivre's Theorem:

$$(\sqrt{3} + i)^6$$

Solution: first write the number in the polar form:

$$\sqrt{3} + i = 2\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right).$$

Since:

$$r = \sqrt{a^2 + b^2} = \sqrt{4} = 2;$$

$$\theta = \tan^{-1} \frac{b}{a} = \tan^{-1} \frac{1}{\sqrt{3}} = \frac{\pi}{6}.$$

Then use De Moivre's Theorem:

$$\begin{aligned}(\sqrt{3} + i)^6 &= [2(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6})]^6 \\ &= 2^6(\cos \pi + i \sin \pi) \\ &= 64(-1 + i \cdot 0) \\ &= -64.\end{aligned}$$