Lecture 4. Hedging in the Futures Market

Mathematics in Hedging

Solutions

Problem 26. The price of gold is currently \$500 per ounce. The forward price for delivery in 1 year is \$700. An arbitrageur can borrow money at 10% per annum. what should the arbitrageur do? Assume that the cost of storing gold is zero and that gold provides no income.

Solution The arbitageur could borrow money to buy 100 ounces of gold today and a short futures contract on 100 ounces of gold for delivery in one year. In particular gold is purchased for \$500 per ounce plus \$50 for interest. The gold is sold at \$700 one year later for a profit of \$150 per ounce, i.e.

$$100 \times (700 - 550) = \$15,000$$

per contract. This is a $\frac{150}{550} = 27.3\%$ return on investment! Buy as much as possible.

Solutions

Problem 27. The current price of a stock is \$94, and 3-month European call options with a strike price of \$95 currently sell for \$4.70. An investor who feels that the price of the stock will increase is trying to decide between buying 100 shares and buying 2,000 call options (=20 contracts). Both straegies involve an investment of \$9,400. What advice would you give? How high does the stock price have to rise for the option strategy to be more profitable?

Solution The investment in call options entails higher risks but can lead to higher returns. If the stock price stays at \$94, an investor who buys call options loses \$9,400 whereas an investor who buys shares neither gains nor loses anything. If the stock price rises to \$120, the investor who buys call options gains

$$2000 \times (120 - 95) - 9400 = \$40,600$$

An investor who buys shares gains

$$100 \times (120 - 94) = \$2,600$$

Solutions

Solution The strategies are equally profitable if the stock price rises to level, S, where

$$100 \times (S - 94) = 2000(S - 95) - 9400,$$

or

$$S = 100$$

The option strategy is more profitable if the stock price rises above $100.

Solutions

Problem 28. On May 29, 2003, an investor owns 100 Intel shares. As indicated in the book, the share price is \$20.83 and an October put option with a strike price of \$20 costs \$1.50. The investor is comparing two alternatives to limit downside risk. The first is to buy 1 October put option contract with a strike price of \$20. The second involves instructing a broker to sell the 100 shares as soon as Intel's price reaches \$20. Discuss the advantages and disadvantages of the two strategies.

Solution The put option costs \$150 and guarantees that the holding can be sold for \$2000 any time up to October. If the stock price falls marginally below \$20 and then rises the option will not be exercised. On the other hand the second alternative (called a stop or stop-loss order) costs nothing and ensures that \$2000 or close to \$2000 is realized for the holding in the event the stock price ever falls to \$20. Thus the stop order will be exercised at the \$20 mark, whereas the put option has some flexibility. For example if the stock price drops to \$19.70 and then rises to \$40, then the put option is better. The investor is paying \$150 for that option.

Solutions

Problem 29. A bond issued by Standard Oil worked as follows. The holder received no interest. At the bond's maturity the company promised to pay $1,000 plus an additional amount based on the price of oil at that time. The additional amount was equal to the product of 170 and the excess (if any) of the price of a barrel of oil at maturity over $25.00. The maximum additional amount paid was $2,550 (corresponds to a price of $40 per barrel). Show that the bond is a combination of a regular bond, a long position in call options on oil with strike price of $25, and a short position in call options on oil with a strike price of $40.

Solution Let S_T denote the price of oil at the bond's maturity. The addition to $1000 the standard oil pays

$$\begin{array}{rl} 0 & S_T < \$25 \\ 170(S_T - 25) & 25 < S_T < 40 \\ 2,550 & 40 < S_T \end{array}$$

Solutions

Solution, cont. This is a payoff from 170 call options on oil with a strike price of 25 less the payoff from 170 call options on oil with a strike price of 40. Thus the the bond is equivalent to a regular bond plus a long position in 170 call options on oil with a strike price of $25 plus a short position in 170 call options on oil with a strike price of $40.

Basis Risk

Hedging not quite straightforward, since:

- The asset whose price is to be hedged may not be exactly the same as the asset underlying the futures contract.
- The hedger may be uncertain as to the exact date when the asset will be bought or sold
- The hedge may require the futures contract to be closed out before its delivery month

These reasons give rise to basis risk

The Basis

We define the basis as

Basis = Spot price of the asset to be hedged − Futures price of contract used

- If the asset being hedged is the same as the the asset underlying the futures contract, the basis should be zero at the expiration of the futures contract.
- Prior to expiration, the basis may be positive or negative. The spot price should equal the futures price for a very short maturity contract
- When the spot price increases by more than the futures prices, the basis increases - called strengthening of the basis.
- When the futures price increases by more than the spot price, the basis declines - called weakening of the basis

Basis Risk

Use the following notation

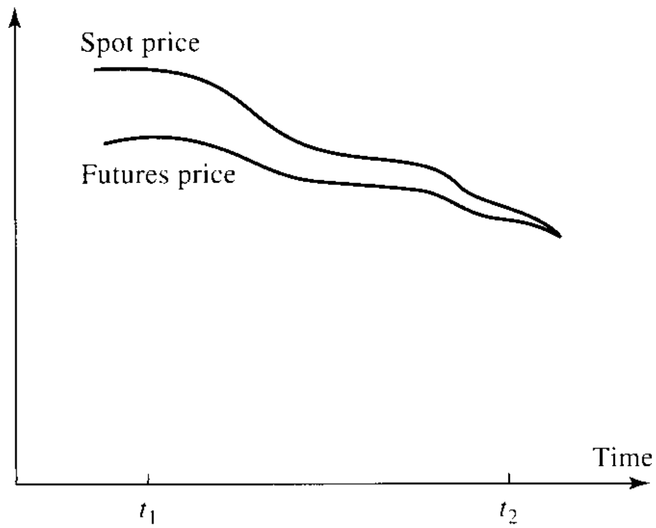
$$
\begin{aligned}
S_1 &\equiv \text{ Spot price at time } t_1 \\
S_2 &\equiv \text{ Spot price at time } t_2 \\
F_1 &\equiv \text{ Futures price at time } t_1 \\
F_2 &\equiv \text{ Futures price at time } t_2 \\
b_1 &\equiv \text{ Basis at time } t_1 \\
b_2 &\equiv \text{ Basis at time } t_2
\end{aligned}
$$

Suppose at the time of initiation the hedge and spot prices are \$2.50 and \$2.20, respectively. At the time of closing of the position, the hedge and spot are \$2.00 and \$1.90, respectively. Hence, $S_1 = 2.50$, $F_1 = 2.20$ and $S_2 = 2.00$, $F_2 = 1.90$.

Basis Risk

Consider the following basis variation over time:

Figure 3.1 Variation of basis over time.



From our definition of basis we have

$$b_1 = S_1 - F_1 \quad \text{and} \quad b_2 = S_2 - F_2$$

or $b_1 = 0.3$ and $b_2 = 0.1$.

Basis Risk, cont.

- Consider a hedger who knows that the asset will be sold at time t_2 and takes a short futures position at time t_1

- The price realized for the asset is S_2 and the profit on the futures position is $F_1 - F_2$. The effective price that is obtained for the asset with hedging is:

$$S_2 + F_1 - F_2 = F_1 + b_2 (= \$2.30).$$

  The value of F_1 is known at t_1, but we do not know b_2. If we did know b_2 then we would be able to construct a perfect hedge. This is an uncertainty in the hedge.

- We call b_2 the basis rate.

- Basis risk can improve or worsen a hedge position

Basis Risk, cont.

Example:

- Consider a *long* hedge.
- If the basis weakens, the hedge position improves - since Spot Price $<$Future Price
- If the basis strengthens, the hedge position worsens - since Spot Price $>$ Future Price
- If the asset being hedged is different than the asset underlying the futures contract, then the basis risk is usually higher. Example: if S_2^* is the spot price of the asset underlying the hedge then

$$S_2 + F_1 - F_2 = F_1 + (S_2^* - F_2) + (S_2 - S_2^*)$$

denotes the profit of the hedge. $S_2^* - F_2$ and $S_2^* - F_2$ denote two parts of the basis.

Choice of Contract

Basis risk is affected by the choice of the futures contract to be hedge. There are two important factors:

- The choice of the asset underlying the futures contract
- The choice of the delivery month

Note that

- if the assets match then the first choice is straightforward.
- the choice of the delivery month can affect the hedge.

Choice of Contract, cont.

In general

- Futures prices during delivery month can be erratic due to possibility of delivery, so better to have a time difference between the two.
- On the other hand as the time difference between hedge expiration and delivery month $\uparrow$, then usually basis risk $\uparrow$.
- Rule of thumb: choose a delivery month that is as close as possible to, but later than, the expiration of the hedge.

Choice of Contract, cont.

Example Consider a US Company on March 1 that expects to receive 50 million Yen at the end of July. Yen contracts on CME have delivery on March, June, September, and December. One contract is for delivery of 12.5 million Yen. The company *shorts* four September yen future contracts on March 1. When the yen are received a the end of July, the company closes out the short position.

Suppose futures price on March 1 in cents per yen is 0.780 and the spot and futures prices are 0.720 and 0.725, respectively when the contracts are closed. Gain on the contract is 0.780-0.725 = 0.055 cents per yen. The basis is 0.720-0.725 = -0.005 cents per yen when the contract is closed. The effective price obtained in cents per yen is the final spot price plus thegain in the futures:

$$0.7200 + 0.055 = 0.7750$$

or

$$0.780 + (-0.005) = 0.775$$

which is initial futures price plus final basis.

Choice of Contract, cont.

Example

- It is June 8th and a company knows that it will need to purchase 20,000 barrels of crude oil at some time in October or November.
- Oil futures contracts are currently traded every month on NYMEX and the contract size is 1000 barrels. The company decides to use the December contract for hedging and takes a long position in 20 December contracts.
- The futures price on June 8 is $18.00 per barrel.
- The company finds that it is ready to purchase the crude oil on November 10. It closes out the futures contracts on that date. The spot price and futures price on that date are $20.00 per barrel and $19.10 per barrel.
- The gain on the contract is 19.10 - 18.00 = $1.10 per barrel. The basis when the contract is closed out is 20.00 - 19.10 = $0.90 per barrel. The effective price paid is

$$\text{initial futures price } + \text{basis} = 18.00 + 0.90 = 18.90$$

Or a total price of $18.90 \times 20,000 = \$378,000$.

Cross Hedging

Suppose now that the asset being hedged is different from the asset underlying the futures contract.

When assets differ, one uses cross hedging. Example,

- Consider an airline that will purchase jet fuel.
- There are future contracts on jet fuel, so the airline chooses to hedge with a short position in heating oil futures. (in detail)

The hedge ratio is the ratio of the size of the position in futures contracts to the size of the exposure.

- When the asset is the same as the futures contract, then the hedge ratio is 1.0.
- Not always optimal to choose the hedge ratio to be 1.0.

Basic Statistics

Consider sequence of number x_j then

$$\overline{x} \equiv \frac{1}{n}\sum_{j=1}^{n} x_j$$

$$\sigma_x \equiv \sqrt{\frac{\sum x_j^2}{n-1} - \frac{\left(\sum x_j\right)^2}{n(n-1)}}$$

are the mean and sample standard deviation. σ_x measures how far a sequence *deviates* from the mean.

Define for two sequences x_j and y_j the sample correlation

$$\rho = \frac{\sum(x_j - \overline{x})(y_j - \overline{y})}{(n-1)\sigma_x\sigma_y} = \frac{n\sum x_j y_j - \sum x_j \sum y_j}{\sqrt{[n\sum x_j^2 - (\sum x_j)^2][\sum y_j^2 - (\sum y_j)^2]}}$$

Basic Statistics, cont.

Standard deviation formula is

$$\sigma = \sqrt{\frac{\sum x_j^2}{n-1} - \frac{\left(\sum x_j\right)^2}{n(n-1)}}.$$

Define the variance $v = \sigma^2$ then for a pair of sequences

$$\begin{aligned}
\sigma_{x-y}^2 &= \frac{\sum \left(x_j - y_j\right)^2}{n-1} - \frac{\left(\sum x_j - y_j\right)^2}{n(n-1)} \\
&= \frac{\sum x_j^2}{n-1} + \frac{\sum y_j^2}{n-1} - \frac{\sum 2x_j y_j}{n-1} \\
&\quad - \frac{\left(\sum x_j\right)^2}{n(n-1)} - \frac{\left(\sum y_j\right)^2}{n(n-1)} + \frac{\left(\sum x_j\right)\left(\sum y_j\right)}{n(n-1)} \\
&= \sigma_x^2 + \sigma_y^2 - 2\rho\sigma_x\sigma_y
\end{aligned}$$

Minimum Variance Hedge Ratio

We set the following:

$\Delta S \equiv$ Change in spot price over the life of the hedge

$\Delta F \equiv$ Change in spot price over the life of the hedge

$\sigma_S \equiv$ Standard deviation of S

$\sigma_F \equiv$ Standard deviation of F

$\rho \equiv$ Coefficient of correlation between ΔS and ΔF

$h^* \equiv$ Hedge ratio that minimizes the variance of the hedger's position

Claim

$$h^* = \rho \frac{\sigma_S}{\sigma_F}$$

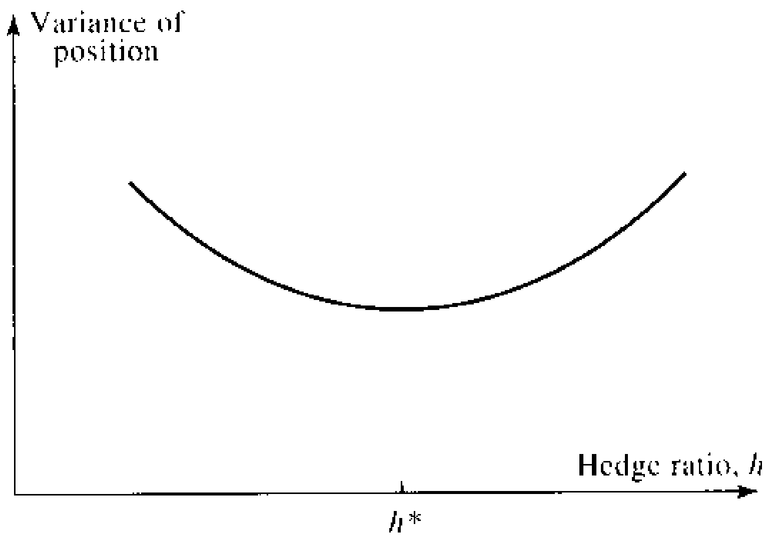
Note that σ_S, σ_F and ρ are estimated from historical data.
Proof of Minimum Variance Hedge Ratio follows later...

Minimum Variance Hedge Ratio

As we saw, choosing

$$h^* = \rho \frac{\sigma_S}{\sigma_F}$$

minimizes the variance of $\Delta S - h\Delta F$.



Minimizing the variance ensures that there will be less risk in using futures with an underlying asset that differs from the original asset.

Minimum Variance Hedge Ratio

- If $\rho = 1$ (completely correlated) and $\sigma_F = \sigma_S$ then $h^* = 1$. In this case the futures price mimics the spot price.

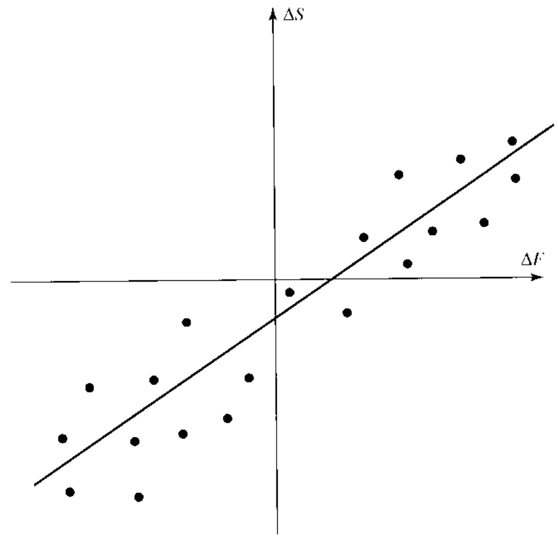
- If $\rho = 1$ and $\sigma_F = 2\sigma_S$ then $h^* = 0.5$. This follows since the futures price always changes by twice the spot price.

- Optimal hedge ratio h^* is the slope of the best-fit line when ΔS is regressed against ΔF.

- We expect there to be a linear relationship between ΔS and ΔF.

Minimum Variance Hedge Ratio, cont.

Regression of change in spot price against change in futures price



- The hedge effectiveness is defined as the proportion of the variance that is eliminated by hedging.

- We define this as ρ^2 (correlation squared) or

$$(h^*)^2 \frac{\sigma_F^2}{\sigma_S^2}$$

Example of Hedge Ratio calculation

Define

$$N_A \equiv \text{ Size of the position being hedged (units)}$$
$$Q_F \equiv \text{ Size of one futures contract (units)}$$
$$N^* \equiv \text{ Optimal number of futures contracts for hedging}$$

- The number of futures contracts should have a value of $h^* N_A$.
- The number of futures contracts required is given by

$$N^* = \frac{\text{value}}{\text{price per contract}} = \frac{h^* N_A}{Q_F}$$

Example, cont.

- An airline expects to purchase 2 million gallons of jet fuel in 1 month and decides to use heating oil futures for hedging.
- Define ΔS to be the jet fuel price per gallon
- Define ΔF to be the futures price for the contract on the heating oil

Month	Change in futures price per gallon	Change in fuel price per gallon
1	0.021	0.029
2	0.035	0.020
3	-0.045	-0.044
4	0.001	0.008
5	0.044	0.026
6	-0.029	-0.019
7	-0.026	-0.010
8	-0.029	-0.007
9	0.048	0.043
10	-0.006	0.011
11	-0.036	-0.036
12	-0.011	-0.018
13	0.019	0.009
14	-0.027	-0.032
15	0.029	0.023

s_F	0.031
s_S	0.026
Rho	0.928
H^*	0.780

Example, cont.

Recall that $h^* = \rho\frac{\sigma_S}{\sigma_F}$ then

$$h^* = .0928\frac{0.0263}{0.0313} = 0.78 < 1$$

The number of contracts should be

$$\frac{0.78 \times 2,000,000}{42,000} = 37.14 \approx 37$$

since there are 42,000 gallons of heating oil per contract.

Proof of Minimum Variance Hedge Ratio Formula

- Expect to sell N_A units of our asset at time t_2 and choose hedge at time t_1 by shorting futures on N_F units of a similar asset. The hedge ratio, h is

$$h = \frac{N_F}{N_A}.$$

Denote Y as the total amount realized for the asset when the profit/loss on the hedge is taken into account. Then

$$\begin{aligned} Y &= \text{spot price} \times \text{amount of asset} \\ &\quad + \text{difference in futures price} \times \text{amount of futures contracts} \\ &= S_2 N_A + (F_1 - F_2) N_F \\ &= S_1 N_A + (S_2 - S_1) N_A - (F_2 - F_1) N_F \\ &= S_1 N_A + (\Delta S) N_A - (h \Delta F) N_A \end{aligned}$$

Proof of Minimum Variance Hedge Ratio Formula

Let v be the variance of $\Delta S - h\Delta F$ then

$$v = \sigma_S^2 + h^2\sigma_F^2 - 2h\rho\sigma_S\sigma_F.$$

Therefore,

$$\frac{dv}{dh} = 2h\sigma_F^2 - 2\rho\sigma_S\sigma_F,$$

and $\frac{dv}{dh} = 0$ when

$$h = \rho\frac{\sigma_S}{\sigma_F}.$$

Finally, we note that $\frac{d^2v}{dh^2} = 2\sigma_F^2 \geq 0$. Therefore, v attains its minimum at this h^*.

Stock Index Futures

A stock index tracks changes in the value of a hypothetical portfolio of stocks.

- Weight of a stock in the portfolio equals the proportion of the portfolio invested in the stock.
- Percentage increase in the stock index over a small period of time is set by the percentage increase in the value of the hypothetical portfolio.
- If one stock rises (drops) sharply, then the stock is given more (less) weight.
- Underlying portfolio is adjusted automatically.

Stock Index futures in the newspaper:

Index Futures

DJ Industrial Average (CBT)-$10 x index

Mar	10446	10507	10418	10440	-38	10687	8580	36,831
June	...	...	...	10419	-38	10475	9000	581

Est vol 11,816; vol Tue 182; open int 37,455, -65.
Idx prl: Hi 10524.22; Lo 10447.18; Close 10470.74, -34.44.

Mini DJ Industrial Average (CBT)-$5 x index

Mar	10446	10506	10417	10440	-38	10687	9069	46,175

Vol Wed 70,499; open int 48,145, -1,739.

DJ-AIG Commodity Index (CBT)-$100 x index

Feb	...	...	...	439.3	-3.5	456.2	452.1	2,351

Est vol 1,150; vol Tue 220; open int 2,571, unch.
Idx prl: Hi 139.159; Lo 137.163; Close 137.350, -1.171.

S&P 500 Index (CME)-$250 x index

Mar	113290	113360	112300	112390	-910	123950	77700	585,763
June	112620	113100	112250	112290	-910	115350	78000	21,212

Est vol 46,110; vol Tue 45,600; open int 610,710, +107.
Idx prl: Hi 1136.03; Lo 1124.74; Close 1126.52, -9.51.

Mini S&P 500 (CME)-$50 x index

Mar	113300	113350	112200	112400	-900	115500	98650	539,366

Vol Wed 595,531; open int 550,820, -18,936.

S&P Midcap 400 (CME)-$500 x index

Mar	584.50	586.00	580.30	580.80	-6.00	603.25	559.75	15,879

Est vol 582; vol Tue 672; open int 15,880, -98.
Idx prl: Hi 587.39; Lo 580.91; Close 581.63, -5.76.

Nasdaq 100 (CME)-$100 x index

Mar	148850	148850	146200	146300	-2400	150900	146200	72,861

Est vol 14,295; vol Tue 9,985; open int 72,918, -246.
Idx prl: Hi 1482.35; Lo 1461.01; Close 1462.61, -29.24.

Mini Nasdaq 100 (CME)-$20 x index

NYSE Composite Index (NYFE)-$50 x index

Mar	...	...	...	6509.50	-57.00	6556.00	6115.00	1,260

Est vol 0; vol Tue 0; open int 1,260, unch.
Idx prl: Hi 6574.76; Lo 6520.91; Close 6526.10, -48.72.

U.S. Dollar Index (FINEX)-$1,000 x index

Mar	87.04	87.30	86.92	87.02	.04	103.18	85.10	16,414
June	...	...	...	87.43	.04	88.37	85.71	2,116

Est vol 2,500; vol Tue 2,272; open int 18,543, +610.
Idx prl: Hi 87.10; Lo 86.70; open int 86.84, +.05.

Nikkei 225 Stock Average (CME)-$5 x index

Mar	10400.	10510.	10360.	10380.	-265	11155.	7670.	30,555

Est vol 3,558; vol Tue 2,468; open int 30,730, +33.
Index: Hi 10627.26; Lo 10418.77; Close 10447.25, -194.67.

Share Price Index (SFE)-AUD 25 x index

Mar	3257.0	3267.0	3250.0	3254.0	-2.0	3346.0	2700.0	160,822
June	3264.0	3278.0	3264.0	3266.0	-2.0	3350.0	2700.0	3,931

Est vol 10,928; vol Tue 10,169; open int 167,890, +2,133.
Index: Hi 3273.5; Lo 3263.6; Close 3265.6, +1.3.

CAC-40 Stock Index (MATIF)-€10 x index

Feb	3626.0	3632.5	3603.0	3614.0	-29.5	3729.5	3531.5	346,178
Mar	3630.0	3634.5	3610.5	3620.0	-29.5	3734.5	2885.0	130,956
June	3563.5	3563.5	3562.5	3560.5	-29.0	3651.5	3282.0	8,810

Est vol 77,301; vol Tue 76,586; open int 489,860, +19,063.
Index: Hi 3625.38; Lo 3602.94; Close 3607.57, -30.64.

Xetra DAX (EUREX)-€25 x index

Mar	4050.0	4056.0	4018.0	4029.5	-31.0	4190.0	3237.5	286,286
June	4065.0	4074.5	4042.5	4050.5	-31.0	4210.0	3251.0	10,167
Sept	4086.5	4096.0	4064.0	4072.0	-31.5	4231.0	3961.0	2,874

Vol Wed 113,473; open int 299,327, -1,522.
Index: Hi 4050.08; Lo 4008.80; Close 4028.37, -29.14.

FTSE 100 Index (LIFFE)-£10 x index

Index quotes list: month, open, high, low, settle, change, lifetime high, lifetime low, and open interest.

Example Stock Indices

- Dow Jones Industrial Average - based on a portfolio of 30 large (blue chip) stocks in the US. Weights are proportional to their stock prices. Two contracts on CBOT. One is $\$10 \times$ index price and the other (Mini DJ) is $\$5 \times$ index price.
- Standard & Poor's 500 - based on a portfolio of 500 stocks: 100 industrial, 40 utilities, 20 transportation, and 40 financial institutions. Weights are based on market capitalization (net worth of companies). Index accounts for 80% of the NYSE value. CME trades two contracts. One is $250 times index and other (Mini S&P) is $50 times index.
- Nasdaq 100 - based on 100 stocks traded on NASDAQ. The CME rades two contracts. One is $100 times index and the other (Mini NASDAQ) is $20 times index.
- Russell 2000 Index - based on 2000 small capitalization stocks in US.
- NYSE Composite Index - based on all stocks traded on NYSE.
- US Dollar Index - trade weighted index of six currencies (euro, yen, pound, Canadian dollar, Swedish krona, Swiss franc)
- Nikkei 225 Stock Average - based on portfolio of 225 of largest stocks on Tokyo Stock Exchange.
- Share Price Index - broadly based index of Australian stocks.
- Etcetera

Hedging an Equity Portfolio

Stock index futures are useful for hedging a well-diversified stock portfolio. Let

$P \equiv$ Current value of the portfolio

$A \equiv$ Current value of the stocks underlying one futures contract

If the portfolio exactly mirrors the index, the optimal hedge ratio $h^* = 1$. Then the number of futures contracts should be

$$N^* = \frac{P}{A}$$

Example: Suppose a portfolio worth \$1 million mirrors the S&P 500. The current value of the index is 1,000, and each futures contract is worth \$250 times the index. Then

$$P = 1,000,000 \qquad A = 250,000.$$

Beta

If the portfolio does not exactly mirror the index, then we use β to determine the appropriate hedge ratio.

- If $\beta = 1$ then the return on the portfolio mirrors the return on the stock index
- If $\beta = 3$ then the return on the portfolio is three times the return on the stock index; hence, it is three times more sensitive to market movements than a portfolio with $\beta = 1.0$.
- In general

$$\beta = h^* \text{ therefore } N^* = \beta \frac{P}{A}$$

  assuming that the maturity of the futures contract is close to the maturity of the hedge (ignoring daily settlement)

Example of β usage

Suppose

$$\begin{aligned}\text{Value of S \& P 500 index} &\equiv 1000 \\ \text{Value of Portfolio} &\equiv \$5,000,000 \\ \text{Risk-free interest rate } &\equiv 4\% \text{ per annum} \\ \text{Dividend yield on index} &\equiv 1\% \text{ per annum} \\ \beta \text{ of portfolio} &\equiv 1.5\end{aligned}$$

- Assume the futures contract on the S&P 500 with 4 months to maturity is used to hedge portfolio over next 3 months
- Assume current futures price of the contract is 1,010
- One contract for delivery is $250 times the index.

Example of β usage, cont.

- We set $A = 250 \times 1000 = 250,000$ (current value of stocks underlying futures contract)

- By $N^* = \beta \frac{P}{A}$ we have $N^* = 1.5 \times \frac{5{,}000{,}000}{250{,}000} = 30$ contracts.

Consider scenarios: S & P index drops to 900 in 3 months and the futures price is 902, then

- Gain from short futures position is

$$30 \times (1010 - 902) \times 250 = 810,000$$

- Loss on the index is 10 %.

- Dividend pays 1% per year or 0.25% in the quarter.

- Index loss plus Dividend yields a loss of -9.75% in the 3-month period.

Example of β usage, cont.

- Risk-free interest rate is approx. 1% per 3 month period.
- Since the $\beta = 1.5$ then pricing yields

$$\text{expect. returns on portfolio} - \text{ Risk-free interest rate}$$
$$= 1.5 \times (\text{return on index} - \text{ risk-free interest rate})$$

- Expect return on portfolio is

$$1.0 + [1.5 \times (-9.75 - 1.0)] = -15.125$$

The expected value of the portfolio (including dividends) is

$$\$5,000,000 \times (1 - 0.1515) = \$4,243,750$$

the end gain on hedge is

$$4,243,750 + 810,000 = \$5,053,750$$