Math 8680 Homework #2 March 25, 2002

1. Using the root system version of the Weyl denominator formula, verify the type C_N WDF on p. 69 of Bressoud.

2. If T is a non-transitive tournament, prove that T has a 3-cycle.

3. Define a graph G whose vertex set is the set of all tournaments on N players. Two tournaments $T - T'$ are connected by an edge if one can be obtained from the other by reversing a 3-cycle. Prove that if $T - T'$, is an edge of G, then the degree of vertex T=degree of vertex T'.

4. Prove that

$$CT \prod_{1 \le i \ne j \le 3} (1 - x_i/x_j)^{a_i} = \binom{a_1 + a_2 + a_3}{a_1, a_2, a_3}.$$

If possible state and prove an n-variable version. What is

$$CT \frac{(1 - z^{A+B})^{A+B}}{z^{AB}(1 - z^A)^A (1 - z^B)^B}?$$

Can you find an n-variable version? (This may be hard.)

5. Bressoud p. 147, 4.3.7

6. Bressoud p. 148, 4.3.10