RECENT RESULTS FOR THE Q-LAGRANGE INVERSION FORMULA

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ABSTRACT. A survey of the q-Lagrange inversion formula is given, including recent work of Garsia, Gessel, Hofbauer, Krattenthaler, Remmel, and Stanton. Some applications to identities of Rogers-Ramanujan type are stated.

1. Introduction.

One of Ramanujan's favorite topics was the expansion of a given function in a series of other functions. For example, given a formal power series $f(x)$ such that $f(0) = 0$ and $f'(0) \neq 0$, one may ask for the coefficients a_k in the expansion

$$x = \sum_{k=1}^{\infty} a_k f(x)^k. \tag{1.1}$$

This question is answered by the *Lagrange inversion formula* [24, §7.32].

Ramanujan's notebooks [23] contain several examples of expansions which can be found from the Lagrange inversion formula, particularly in Chapters 3 and 9. Even though Ramanujan was aware of the Lagrange inversion formula, he had his own form of it, which Berndt [6] has called Ramanujan's Master Theorem. It is clear from Cauchy's integral theorem that a contour integral can be given for a_k. Ramanujan gave a real integral instead.

In this paper we shall survey recent work on q-analogues of Lagrange's Theorem. We also show how these analogues are related to q-series and the Rogers-Ramanujan identities.

The following notation will be used for formal power series. The coefficient of x^k in $g(x), f(x), F(x)$, etc., will be denoted g_k, f_k, F_k. If these coefficients are functions of q they will be denoted $f_k(q)$. In §2 all formal power series have complex coefficients; in later sections the coefficients are rational functions of q. Let $\langle x^n | F(x)\rangle$ denote the coefficient of x^n in $F(x)$. A formal Laurent series is a formal power series plus a finite number of negative integral powers. Let $\underset{x}{Res} F(x)$ denote the coefficient of x^{-1} in a formal Laurent series $F(x)$. Thus for any formal power series $F(x)$

$$\langle x^n | F(x)\rangle = \underset{x}{Res} \frac{F(x)}{x^{n+1}}.$$

We also adopt the usual notation from q-series,

$$\begin{aligned}
(a;q)_k = (a)_k &= \prod_{m=1}^{k} (1 - aq^{m-1}),\\
(a, b, c, \cdots)_k &= (a)_k (b)_k (c)_k \cdots ,\\
k!_q &= (q)_k/(1-q)^k,\\
[k] &= (1-q^k)/(1-q)
\end{aligned}$$

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and the q-binomial coefficient

$$\binom{n}{k}_q = \frac{(q)_n}{(q)_k (q)_{n-k}}.$$

2. Lagrange's Theorem.

Not only can the coefficients a_k of the inverse function to $f(x)$ in (1.1) be found, but also the coefficients for any formal power series $F(x)$ in x. This is Lagrange's Theorem.

Theorem 1 (Lagrange inversion formula). *Let $f(x)$ be a formal power series with $f(0) = 0$ and $f'(0) \neq 0$. For any formal power series $F(x)$, if*

$$F(x) = \sum_{k=0}^{\infty} a_k f(x)^k \tag{2.1}$$

then

$$a_k = \operatorname*{Res}_x \frac{F(x) f'(x)}{f(x)^{k+1}} \tag{2.2}$$

$$= \operatorname*{Res}_x \frac{F'(x)}{k f(x)^k}. \tag{2.3}$$

Proof. Equation (2.2) follows from

$$\begin{aligned} \operatorname*{Res}_x (f(x)^{k-j} f'(x)) &= \operatorname*{Res}_x (f(x)^{k-j+1})'/(k-j+1) \\ &= \begin{cases} 0 \text{ for } & j \neq k+1, \\ 1 \text{ for } & j = k+1. \end{cases} \end{aligned} \tag{2.4}$$

Equation (2.2) follows from (2.3) and

$$\operatorname*{Res}_x \left(\frac{F(x)}{f(x)^k} \right)' = 0. \quad \square$$

It is clear that if $F(x)$ is replaced by $F(x)/f'(x)$, then (2.2) simplifies. This is sometimes called the second form of the Lagrange inversion formula.

Theorem 2 (Second form of the Lagrange inversion formula). *Let $f(x)$ be a formal power series with $f(0) = 0$ and $f'(0) \neq 0$. Then for any formal power series $F(x)$, if*

$$F(x)/f'(x) = \sum_{k=0}^{\infty} a_k f(x)^k$$

then

$$a_k = \operatorname*{Res}_x \frac{F(x)}{f(x)^{k+1}}. \tag{2.5}$$

A classical example of these two theorems is Abel's theorem

$$e^{ax} = \sum_{k=0}^{\infty} \frac{a(a-bk)^{k-1}}{k!} x^k e^{kbx} \tag{2.6}$$

and

$$e^{(a-b)x}/(1+bx) = \sum_{k=0}^{\infty} \frac{(a-b(k+1))^k}{k!} x^k e^{kbx}. \tag{2.7}$$

Frequently Theorems 1 and 2 are stated in another way. First replace x by the functional inverse $f^{\langle -1\rangle}(x)$, to obtain

$$F(f^{\langle -1\rangle}(x)) = \sum_{k=0}^{\infty} a_k x^k \tag{2.8}$$

where a_k is given by (2.2). For $F(x) = x^n$, (2.8) is

$$f^{\langle -1\rangle}(x)^n = \sum_{k=0}^{\infty} a_k x^k. \tag{2.9}$$

Next let $x/R(x) = g(x)$, and put $g(x) = f^{\langle -1\rangle}(x)$ so that $g(x)$ satisfies

$$g(x) = xR(g(x)).$$

Then (2.2) becomes

$$\langle x^k | g(x)^n\rangle = \frac{k}{n}\langle x^{n-k} | R(x)^k\rangle. \tag{2.10}$$

A calculation shows that (2.4) becomes, if $g(x) = xR(g(x))$,

$$\langle x^k | \frac{g(x)^n}{1-xR'(g(x))}\rangle = \langle x^{k-n} | R(x)^k\rangle. \tag{2.11}$$

3. q-Lagrange inversion.

There have been two different approaches, each with its own goal, to the q-Lagrange inversion problem. The first approach is the most natural: write down a q-analogue of (2.1) and then give some formula for the coefficients a_k which is a q-analogue of (2.2) or (2.3). Of course to be a satisfactory solution, the q-analogue to (2.1) must be reasonable, and the resulting formula for a_k must be simple enough to be useful. In particular any such general theorem should easily reproduce the known examples of q-Lagrange inversion. Unfortunately these two goals have not simultaneously been met, and the second approach, in lieu of the first, is necessary. Find non-trivial and important families of q-Lagrange inversion.

Let $f(x, q)$ be a formal power series in x, with coefficients that are rational functions of q. Sometimes we shall suppress q and write $f(x)$ instead of $f(x, q)$. As usual, we can assume that $f_0(q) = 0$ and $f_1(q) = 1$. A good candidate for the q-analogue of the inverse to $f(x)$ is a formal power series $\phi(x)$ such that

$$\sum_{k=1}^{\infty} f_k(q)\phi(x)\phi(xq)\cdots\phi(xq^{k-1}) = x. \tag{3.1}$$

Clearly (3.1) is a q-analogue of $f(\phi(x)) = x$, and the relation $\phi(f(x)) = x$ could become

$$\sum_{k=1}^{\infty} \phi_k(q)f(x)f(xq^{-1})\cdots f(xq^{1-k}) = x. \tag{3.2}$$

Garsia [10] has shown that these two formulations of a q-analogue of a functional inverse are equivalent.

Theorem 3 (Garsia). *Equation (3.1) holds if, and only if, (3.2) holds.*

Andrews [2], Gessel [13], and Garsia [10] have each given versions of a general form of q-Lagrange inversion. Good expositions of these papers are given in [10], [17], or [19]. They were motivated by an example of Carlitz [8], who gave a q-analogue of Theorem 1 for the function $f(x) = x/(1-x)$. He replaced (2.1) by

$$F(x) = \sum_{k=0}^{\infty} a_k \frac{x^k}{(1-x)(1-xq)\cdots(1-xq^{k-1})}. \tag{3.3}$$

It is clear that a general form of (3.3) for $f(x) = x/r(x)$ is

$$F(x) = \sum_{k=0}^{\infty} a_k \frac{x^k}{r(x)r(xq)\cdots r(xq^{k-1})}. \tag{3.4}$$

Andrews [2] gave a general formula for the coefficients a_k as a determinant in the coefficients F_k of $F(x)$, but his result is very difficult to apply.

Garsia [10] has a very elegant form of q-analogue of Theorem 1 for the expansion of $F(f^{\langle -1\rangle}(x))$ in (2.7). Suppose that $f(x)$ and $\phi(x)$ satisfy (3.1) and (3.2). Then we are expanding

$$\sum_{k=0}^{\infty} F_k(q)\phi(x)\phi(xq)\cdots\phi(xq^{k-1}) = \sum_{k=0}^{\infty} a_k x^k. \tag{3.5}$$

Garsia's analogue of (2.2) for a_k involves a miraculous q-analogue of the derivative $f'(x)$, which he called ${}^\circ f(x)$. Again let $f(x) = x/r(x)$ and define the "roofing" and "starring" operators by

$${}^*r(x) = \prod_{m=0}^{\infty} r(xq^{-m}), \tag{3.6}$$

$$\check{r}(x) = \sum_{k=0}^{\infty} r_k(q) q^{\binom{k}{2}} x^k. \tag{3.7}$$

Then ${}^\circ f(x)$ is defined by

$${}^\circ f(x) = {}^*r(xq^{-1}) \left(\frac{1}{{}^*(\check{r}(xq))} \right)\check{}. \tag{3.8}$$

It is not at all obvious that (3.8) is a q-analogue of $f'(x)$.

Theorem 4 (Garsia's q-Lagrange inversion formula). *Let $f(x)$ and $\phi(x)$ satisfy (3.1), and $F(x)$ satisfy (3.5). Then*

$$a_k = \operatorname*{Res}_x \frac{{}^\circ f(xq^{-k})F(x)}{f(x)\cdots f(xq^{-k})},$$

where ${}^\circ f(x)$ is defined by (3.8).

Moreover Garsia shows that

$$\sum_{k=0}^{\infty} F_k(q)\phi(x)\phi(xq)\cdots\phi(xq^{k-1}) = \frac{(F(x)\,{}^*r(x))\check{}}{({}^*r(x))\check{}} \tag{3.9}$$

which corresponds to no known result for $q = 1$.

Gessel [13, Th. 6.9] gave a q-analogue of the alternative (2.11) to Theorem 2. He replaced the functional equation $f(x) = xR(f(x))$ by

$$f(x,q) = qx \sum_{k=0}^{\infty} R_k(q) f(x,q)f(xq,q)\cdots f(xq^{k-1},q), \tag{3.11}$$

and found the next theorem. We shall use the notation

$$f^{[k]}(x,q) = f(x,q)f(xq,q)\cdots f(xq^{k-1},q). \tag{3.12}$$

Theorem 5 (Gessel's q-Lagrange inversion formula). *Let $R(x)$ and $f(x)$ satisfy (3.11). Then*

$$\langle x^k | f^{[k]}(x,q)/(1-xd(x))\rangle = q^{n(n+1)/2}\langle x^{k-n}|R^{[k]}(x,q^{-1})\rangle.$$

where

$$d(x) = \sum_{i,j=0}^{\infty} R_{i+j+1}(q) f^{[i]}(x,q) f^{[j]}(x,q^{-1}).$$

Note that the denominator of (2.10) has been replaced by a double sum in Theorem 3. Garsia [10, Theorem 2.5] gave another form of the denominator using his q-analogue of the derivative (3.9).

4. More q-Lagrange inversion.

Cigler [9], Hofbauer [16], Krattenthaler [19], and Paule [20] have another approach to q-Lagrange inversion for special families of functions. They replaced $f^k(x)$ by a function $x^k/r_k(x,q)$ instead of $x^k/r(x)r(xq)\cdots r(xq^{k-1})$. Naturally unless the function $r_k(x,q)$ has some properties mimicking $r^k(x)$, there is no hope for an explicit formula for a_k in the expansion

$$F(x) = \sum_{k=0}^{\infty} a_k \frac{x^k}{r_k(xq,q)}. \tag{4.1}$$

We may assume that $r_k(0,q) = 1$.

The key property is a q-analogue of (2.4), which for $f(x) = x/r(x)$ is

$$\operatorname*{Res}_x \frac{r(x)^{n-k}}{x^{n+1-k}}\left(1 - \frac{xr'(x)}{r(x)}\right) = \begin{cases} 0 \text{ for } & k \neq n, \\ 1 \text{ for } & k = n. \end{cases}$$

A q-analogue could be

$$\operatorname*{Res}_x \frac{r_n(x,q)}{r_k(xq,q)x^{n+1-k}}\,(1 - x\rho(x)) = \begin{cases} 0 \text{ for } & k \neq n, \\ 1 \text{ for } & k = n, \end{cases} \tag{4.2}$$

where $\rho(x)$ is some q-analogue of $r'(x)/r(x)$. In fact, (4.2) holds if $r_k(x)$ has the following property:

$$D_q r_k(x) = [k]\rho(x) r_k(x) \text{ for all } k \geq 0, \tag{4.3}$$

where D_q is the q-derivative

$$D_q f(x) = \frac{f(xq) - f(x)}{(q-1)x}.$$

To prove (4.2), just compute

$$\operatorname*{Res}_x D_q\left(\frac{r_n(x)}{r_k(x)x^{n-k}}\right) = 0.$$

Theorem 6 (Hofbauer's q-Lagrange inversion). *Let $F(x)$ satisfy (4.1), where $r_k(x)$ satisfies (4.3). Then*

$$a_k = \operatorname*{Res}_x \frac{F(x) r_k(x)(1 - x\rho(x))}{x^{k+1}}.$$

Proof. Equation (4.2) immediately gives the theorem. □

Krattenthaler [19] generalized Theorem 6 by allowing simultaneously two different analogues of the denominator $r^k(x)$. Suppose that $r_k(x,q)$ and $s_k(x,q)$ satisfy $r_k(0,q) = 1$, $s_k(0,q) = 1$, and

$$\begin{aligned} D_q r_a(x) &= [k]\rho(x) r_a(x) \text{ for all real } a, \\ D_q s_b(x) &= [k]\sigma(x) s_b(x) \text{ for all real } b. \end{aligned} \tag{4.4}$$

Theorem 7 (Krattenthaler's q-Lagrange inversion). *Let $F(x)$ satisfy*

$$F(x) = \sum_{k=0}^{\infty} a_k \frac{x^k s_{-k-b}(x,q)}{r_{k+a}(xq,q)}.$$

where $r_a(x)$ and $s_b(x)$ satisfy (4.4). Then

$$a_k = \underset{x}{Res} F(x) \frac{r_{k+a}(x)}{x^{k+1} s_{-k-b}(qx)} (1 - x\rho(x) - x\sigma(x) + x^2\rho(x)\sigma(x)(1-q^{a-b})).$$

Proof. The proof is similar to the proof of Theorem 6, by establishing the appropriate version of (4.2) (see [19, Lemma 1]). □

Krattenthaler also gave a nice version of (2.3) for the $a = b = 0$ special case of Theorem 7

$$a_k = \frac{1}{[k]} \underset{x}{Res} \frac{D_q(F(x)) r_k(x)}{s_{-k}(x) x^k}. \tag{4.5}$$

Note that the (4.5) also gives a version of (2.3) for Theorem 6.

Paule has given a generalization of Theorem 6 [20, Theorem 4].

5. Even more q-Lagrange inversion.

Gessel and Stanton [14], [15] took the point of view that the Lagrange inversion theorem is a matrix inversion result. Specifically, if

$$f^k(x) = \sum_{n=k}^{\infty} B_{nk} x^n \tag{5.1}$$

then (2.1) holds if, and only if, the matrix equation $Ba = f$. holds. The Lagrange inversion theorem gives a formula for the coefficients a_k, $a = B^{-1}f$, thus gives the inverse matrix B^{-1}.

They took a special family of functions replacing $f^k(x)$, namely $x^k/(1-x)^{a+(b+1)k}$, so that

$$B_{nk} = \prod_{j=1}^{n-k} (a + (b+1)k + j - 1)/(n-k)!. \tag{5.2}$$

Theorem 1 implies

$$B^{-1}_{km} = \prod_{j=1}^{k-m-1} (1 - a - (b+1)k + j)(-a - (b+1)m)/(k-m)!. \tag{5.3}$$

The q-analogue of Lagrange inversion for $x/(1-x)^{b+1}$ is given by q-analogues of the matrices in (5.2) and (5.3).

Theorem 8 (q-Lagrange inversion for $x/(1-x)^{b+1}$). *Let*

$$G_k(x) = \sum_{n=k}^{\infty} B_{nk} x^n,$$

where

$$B_{nk} = q^{-nk} (Aq^k p^k; p)_{n-k} / (q)_{n-k}.$$

Then

$$F(x) = \sum_{k=0}^{\infty} a_k G_k(x)$$

if, and only if,

$$a_k = \sum_{m=0}^{k} (Aq^k p^{k-1}; p^{-1})_{k-m-1}(1 - Ap^m q^m) q^{(k^2+m^2+k-m)/2} F_m(q)/(q)_{k-m}.$$

Garsia and Remmel [11] have yet another q-Lagrange inversion formula. They replaced $f_k(x)$ by a q-analogue of $(g(x)-1)^k$,

$$\sum_{s=0}^{k} \binom{k}{s}_q q^{\binom{s}{2}} (-1)^s g(x)g(xq)\cdots g(xq^{k-s-1}). \tag{5.4}$$

Let B_{nk} be the coefficient of x^n in (5.4). They give an explicit formula for the inverse matrix B^{-1}.

Theorem 9 (Garsia and Remmel's q-Lagrange inversion). *Let $G_k(x)$ be defined by (5.4), where $g(0) = 1$. Then*

$$F(x) = \sum_{k=0}^{\infty} a_k G_k(x)$$

if, and only if,

$$a_k = \sum_{m=0}^{k} A_{km} F_m(q),$$

where

$$A_{km} = q^{-\binom{k}{2}} \sum_{s=m}^{k} \frac{(-1)^s q^{\binom{k-s}{2}} \theta_{s-m}}{(q)_s (q)_{k-s} \theta_s}$$

and

$$\theta(x) = 1/\prod_{n=0}^{\infty} g(xq^n).$$

6. Two classical examples.

The first example of q-Lagrange inversion formula was given by Jackson [18, Eq.(5)], who gave a q-analogue of Abel's theorem (2.6). It is

$$E_q(ax) = \sum_{k=0}^{\infty} \frac{a(a-[2]b)\cdots(a-[k]b)}{k!_q} x^k E_q([k]bxq^{1-k}), \tag{6.1}$$

where $E_q(x)$ is a q-analogue of the exponential function

$$E_q(x) = (x(1-q))_\infty. \tag{6.2}$$

Carlitz's example [8] is a q-Lagrange inversion formula for $f(x) = x/(1-x)$. If $F(x)$ satisfies (3.3), then he gave

$$a_k = \frac{1}{[k]} \operatorname*{Res}_x \frac{D_q(F(x))(1-x)\cdots(1-xq^{k-1})}{x^k}. \tag{6.3}$$

7. Comparisons.

In this section the strengths and weaknesses of the various approaches will be given.

First we consider to which functions $f_k(x)$ the q-Lagrange inversion theorems apply. The Garsia-Gessel Theorems and the Garsia-Remmel Theorem apply to all of the corresponding analogues of $f^k(x)$. The Hofbauer-Krattenthaler Theorems apply to a quotient of functions satisfying (4.4). One may ask which functions $r_k(x) = r(x)r(xq)\cdots r(xq^{k-1})$ satisfy (4.3). An easy calculation shows

$$D_q r_k(x) = [k] r_k(x) \frac{D_{q^k} r(x)}{r(x)},$$

so that (4.3) holds, if, and only if,

$$\frac{D_{q^k} r(x)}{r(x)} = \rho(x), \text{ for all } k \geq 0. \tag{7.1}$$

Clearly a linear function is the only solution to (7.1), and we then obtain Carlitz's example.

Next we consider for which classical functions the inversion formulas given can be explicitly computed. The Garsia-Gessel Theorems give only Carlitz's example. There are applications to continued fractions, however, because the functional equations are appropriate. Garsia has used his roofing and starring operators to give new proofs of Rogers-Ramanujan type identities. He did not find any new such identities. His roofing and starring operators certainly deserve more attention. The Garsia-Remmel Theorem applies to $e^x - 1$, and $(1-x)^a - 1$. Hofbauer's Theorem includes Jackson's and Carlitz's examples, while Krattenthaler's Theorem also gives the $b = 1$ and $b = -1/2$ examples of Gessel-Stanton. The $b = 1$ case is particularly important. The entire theory of basic hypergeometric series can be based upon this case. As Andrews has shown [3], the Rogers-Ramanujan identities follow from this case, and it has led to the idea of the Bailey lattice [1]. Recent work of Gasper [12] and Rahman [21] indicates that many applications of bibasic identities (as in Theorem 8) to basic hypergeometric series remain.

8. Rogers-Ramanujan identities.

Consider the Rogers-Ramanujan continued fraction

$$\phi(x) = \cfrac{x}{1 - \cfrac{xq}{1 - \cfrac{xq^2}{\ddots}}}. \tag{8.1}$$

Garsia [10] has shown that the evaluation of (8.1) as a quotient of Rogers-Ramanujan series follows from (3.9). It is easy to see that the defining relation for the continued fraction is

$$\phi(x) - \phi(x)\phi(xq) = x, \tag{8.2}$$

so that we may take $f(x) = x - x^2$ in (3.1). Then (3.9), with $r(x) = 1/(1-x)$ and $F(x) = x$ easily gives the evaluation. In fact Garsia proves the key identity for the Rogers-Ramanujan identities that Rogers and Ramanujan [22] had. Several other examples of continued fractions are given by Gessel [13].

We now give some examples of new identities of Rogers-Ramanujan type that were found by q-Lagrange inversion in [14, Eq.(7.13) and (7.15)]. The first two examples are closely related,

$$1 + \sum_{n=1}^{\infty} \frac{(-q)_{n-1}}{(q)_n} q^{\binom{n+1}{2}} = \frac{1}{(q;q^2)_\infty (q^4, q^6, q^8, q^{10}, q^{22}, q^{24}, q^{26}, q^{28}; q^{32})_\infty} \tag{8.3}$$

and

$$\sum_{n=1}^{\infty} \frac{(-q)_{n-1}}{(q)_n} q^{\binom{n+1}{2}} = \frac{q}{(q;q^2)_\infty (q^2,q^8,q^{12},q^{14},q^{18},q^{20},q^{24},q^{30};q^{32})_\infty}. \tag{8.4}$$

Since the left sides of (8.3) and (8.4) differ by one, Andrews [4, 5] noted that (8.3) and (8.4) imply the next theorem.

Theorem 10. *The number of partitions of* n *into parts which are odd or congruent to* ± 4, ± 6, ± 8, *or* ± 10 *modulo 32 is equal to the number of partitions of* $n-1$ *into parts which are odd or congruent to* ± 2, ± 8, ± 12, *or* ± 14 *modulo 32.*

Andrews also gave a combinatorial interpretation for (8.3) and (8.4) individually as a "colored" Rogers-Ramanujan identity. We state here only the version for (8.3). A *two-color partition* is an ordered pair of partitions (λ, μ), which are called *red* and *green* respectively. Such a two-color partition is called a partition of n if the sum of the parts of λ and μ is n.

Theorem 11. *The number of two-color partitions of* n *such that*

(1) *the parts are distinct,*
(2) *the largest part is red, and*
(3) *each green part is at least two smaller than the next largest part*

is equal to the number of partitions of n *into parts which are odd or congruent to* ± 4, ± 6, ± 8, *or* ± 10 *modulo 32.*

Finally we give Bressoud's [7] combinatorial interpretation of [14, Eq. 7.24]

$$\sum_{k=0}^{\infty} \frac{(-q;q^2)_{2k} q^{2k^2}}{(q^2;q^4)_k (q^8;q^8)_k} = (-q^2,-q^3,-q^5;q^8)_\infty.$$

Theorem 12. *The number of partitions of* n *into distinct parts whose odd parts are congruent to* ± 3 *modulo 8 is equal to the number of partitions of* n *with the following properties:*

(1) *the parts which are congruent to 2 modulo 4 are 2, 6, ...,* $4k-2$, *with multiplicity at least one,*
(2) *the parts which are congruent to 0 modulo 4 are* $\leq 4k$ *and have even multiplicities,*
(3) *all of the odd parts are distinct and less than* $4k$.

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