

Homework #2

Cameron

1. Chap. 4, problem on the inverse of a formal power series with constant term 1.
2. Chap. 4, problem on the Bernoulli numbers recurrence and generating function.

Balakrishnan 2.161, 2.164, 2.171, 3.89, 3.91, 3.102, 3.104, 3.105, 3.107

Supplement

Ordinary generating functions

1. How many ways are there to distribute n identical balls into 4 distinguishable boxes if
 - (a) there are no restrictions,
 - (b) the first box must contain at least 7 balls, and the third box must contain at most 6 balls,
 - (c) the second box must contain an even number of balls, the third box either 0 or 1 ball,
 - (d) the first two boxes contain an equal number of balls.
2. How many integers between 0 and 9999 have a sum of digits equal to 13? Explain why the answer is not $\binom{16}{3}$ and find the correct answer.
3. Show that the answer to problem 2(c) on the last supplementary problem is the coefficient of $x^4 q^{23}$ in

$$\prod_{i=1}^{20} (1 + xq^i)$$

divided by $\binom{20}{4}$.

4. Let a_n be the number of ways to make change for n cents, given an unlimited number of pennies, nickels, and quarters. Find

$$A(x) = \sum_{n=0}^{\infty} a_n x^n.$$

Exponential generating functions

5. How many words w of 0's 1's 2's and 3's have length n and
 - (a) no 2's,
 - (b) at least one 3,
 - (c) at least one 2 and an even number of 1's,
 - (d) an equal number of 0's and 1's?