Homework 1 Solutions

1.8 (a)

$$c_0 + c_1 x + c_2 y + c_3 z + c_4(x^2 - y^2) + c_5(x^2 - z^2) + c_6 xy + c_7 xz + c_8 yz,$$
where $c_0, \ldots, c_8$ are arbitrary constants.

1.7

$u = \log\left[c(x-a)^2 + c(y-b)^2\right]$, for a, b, c arbitrary constants.

1.13

$u = a + \dfrac{b}{r} = a + \dfrac{b}{\sqrt{x^2+y^2+z^2}}$, where a, b are arbitrary constants.

1.20

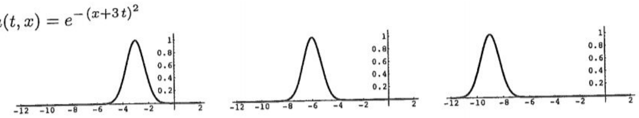
Solution: (a) $\cos(x-2t) + \frac{1}{4}\cos x - 5\sin(x-2t) - \frac{5}{4}\sin x$; (b) $-\sin 3\ \cos(x-2t) - \frac{1}{4}\sin 3\ \cos x + \cos 3\ \sin(x-2t) + \frac{1}{4}\cos 3\ \sin x = \sin(x-2t-3) + \frac{1}{4}\sin(x-3)$.

1.27

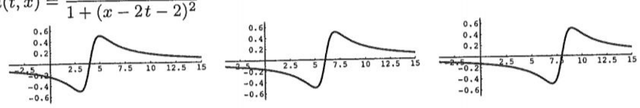
(b) $u(x) = \frac{1}{6}e^x \sin x + c_1 e^{2x/5}\cos\frac{4}{5}x + c_2 e^{2x/5}\sin\frac{4}{5}x$.

2.2.2

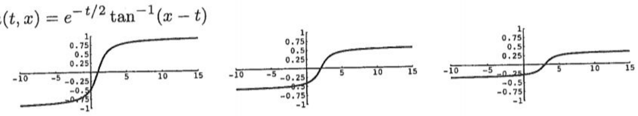
(a) $u(t,x) = e^{-(x+3t)^2}$



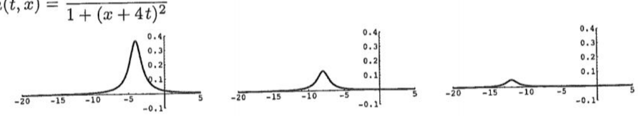
(b) $u(t,x) = \dfrac{x-2t-2}{1+(x-2t-2)^2}$



(c) $u(t,x) = e^{-t/2}\tan^{-1}(x-t)$

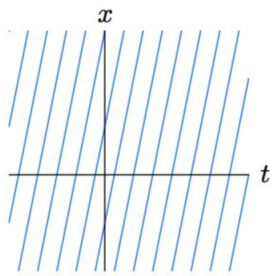


(d) $u(t,x) = \dfrac{e^{-t}}{1+(x+4t)^2}$

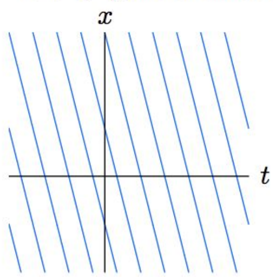


2.2.3

(b) Characteristic lines: $x = 5t + c$; general solution: $u(t,x) = f(x - 5t)$;



(d) Characteristic lines: $x = -4t + c$; general solution: $u(t,x) = e^{-t} f(x + 4t)$;



2.2.5

Solution: $u(t,x) = -\frac{1}{2}\cos x + \frac{1}{2}\cos(x - 2t) + \sin(x - 2t)$.

2.2.9

. (a) $|\, u(t,x) \,| = |\, f(x - ct) \,|\, e^{-at} \leq M\, e^{-at} \to 0$ as $t \to \infty$ since $a > 0$.

(b) For example, if $c \geq a$, then the solution $u(t,x) = e^{(c-a)t - x} \not\to 0$ as $t \to \infty$.