1. True/False (24 points, 2 each). No partial credit.

   (a) **(TRUE)** The Majority Criterion says "A candidate who gets more than 50% of the votes should be elected."

   (b) **(FALSE)** The Condorcet Criterion says "A candidate must win every head-to-head comparison with other candidates in order to win."

   (c) **(TRUE)** The Monotonicity Criterion says "If candidate X is the winner of an election and, in a reelection, the only changes in the ballots are changes that favor X, then X should remain the winner of the election."

   (d) **(TRUE)** If we are having a vote between only two candidates, there is really only one way to decide who is the winner.

   (e) **(FALSE)** In a weighted voting system, someone with more votes automatically has more power.

   (f) **(TRUE)** The quota in a weighted voting system needs to be at least 50% of the total votes.

   (g) **(TRUE)** A dictator automatically has 100% power according to the Banzhaf power index.

   (h) **(TRUE)** Two players with the same number of votes automatically have the same power.

   (i) **(TRUE)** When dividing goods among four people, a player has a fair share if they get a piece that they think is worth at least 25% of the total.

   (j) **(TRUE)** When we are doing fair division, we assume that no players know anything about what the other players think is fair.

   (k) **(FALSE)** The lone-chooser method is used for deciding how to distribute goods that cannot be divided into smaller pieces.

   (l) **(TRUE)** In the divider-chooser method, the divider must cut the goods into two equal pieces.

2. Multiple choice (5 points). No partial credit.

   (a) (2 points) In a weighted voting system, a *dummy* is:

A	Someone whose vote never affects the outcome	B	A non-critical player in a coalition
C	Someone with zero votes	D	None of the above

   The correct answer is A.

   (b) (3 points) Which voting system does not need a preference ballot?

A	The pairwise comparison method	B	The Borda count method
C	The plurality-with-elimination method	D	None of the above

   The correct answer is D. All of these methods require a preference ballot.

3. (15 points) The following questions all refer to this preference ballot, which is the outcome of an election between 4 candidates, U, V, W, and X.

9	10	7	3	6
U	V	W	U	X
V	X	X	W	U
X	U	U	V	W
W	W	V	X	V

(a) (2 points) Rank these candidates using the extended plurality method.

First _U_ Second _V_ Third _W_ Fourth _X_

We rank them according to who got the most first-place votes.

(b) (3 points) Rank these candidates using the *recursive* plurality method.

First _U_ Second _V_ Third _X_ Fourth _W_

U got the most first-place votes (12). We then eliminate U, and find that V now has the most first-place votes (19). We then eliminate V, and find that X has the most first-place votes (25).

(c) (5 points) Rank these candidates using the recursive plurality-with-elimination method.

First _U_ Second _V_ Third _W_ Fourth _X_

Using the plurality-with-elimination method, X is eliminated first with the fewest first-place votes (6), then W is eliminated next (7), then V (10), leaving U.

(d) (2 points) Does this election have a majority candidate? If so, who?

This election does not have a majority candidate, as nobody has more than 50% of the total votes; that would mean having more than $35/2 = 17.5$ first-place votes.

(e) (3 points) Who wins in a head-to-head comparison between U and X?

In a head-to-head comparison, U has 12 votes while X has 23, so X is the winner.

4. (16 points) Weighted voting.

   The following three questions refer to the weighted voting system [10;5,3,3,3].

   (a) (3 points) List all the winning coalitions.

   The winning coalitions are: $\{P_1, P_2, P_3\}$ **(11 votes),** $\{P_1, P_2, P_4\}$ **(11 votes),** $\{P_1, P_3, P_4\}$ **(11 votes), and** $\{P_1, P_2, P_3, P_4\}$ **(14 votes).**

   (b) (3 points) How many times is each player critical?

   P_1 _4_ P_2 _2_ P_3 _2_ P_4 _2_

   P_1 **is always critical, whereas the other 3 are only critical in the coalitions with 3 people.**

   (c) (3 points) Find the Banzhaf power index for every player in this voting system. (You can leave your answer as a fraction.)

   P_1 _40%_ P_2 _20%_ P_3 _20%_ P_4 _20%_

   There are 4 + 2 + 2 + 2 = 10 total criticals, so for each player we divide the number of criticals by 10.

   The following two questions refer to the weighted voting system [4;2,2,1].

   (d) (4 points) List all the sequential coalitions and identify the pivotal player in each.

   There are 6 sequential coalitions:

   - $\langle P_1, P_2, P_3 \rangle$ **-** P_2 **is critical**
   - $\langle P_1, P_3, P_2 \rangle$ **-** P_2 **is critical**
   - $\langle P_2, P_1, P_3 \rangle$ **-** P_1 **is critical**
   - $\langle P_2, P_3, P_1 \rangle$ **-** P_1 **is critical**
   - $\langle P_3, P_1, P_2 \rangle$ **-** P_2 **is critical**
   - $\langle P_3, P_2, P_1 \rangle$ **-** P_1 **is critical**

   (e) (3 points) Find the Shapley-Shubik power indices of all three players in this voting system. (You can leave your answer as a fraction.)

   P_1 _50%_ P_2 _50%_ P_3 _0%_

   P_1 **and** P_2 **are critical 3 times each, and** P_3 **is never critical. We divide these by 6, the number of sequential coalitions.**

5. (20 points) One more.

   (a) (3 points) We are trying to divide a cake between four people using the lone-divider method. Chooser 1 thinks that piece 1 is worth 10% of the cake, piece 2 is worth 20%, piece 3 is worth 45% and piece 4 is worth 25%. What would this chooser's bid be?

   This chooser bids on all the pieces that they think are worth 25% or more, so they would bid on pieces 3 and 4.

   (b) (3 points) In this same division, Chooser 2 bid on pieces 1 and 2, and Chooser 3 bid on pieces 2 and 4. Describe a fair division of this cake.

   There are four ways to divide this cake fairly, any one of the following is correct:

   - **Chooser 1 gets piece 3, Chooser 2 gets piece 1, Chooser 3 gets piece 2.**
   - **Chooser 1 gets piece 3, Chooser 2 gets piece 1, Chooser 3 gets piece 4.**
   - **Chooser 1 gets piece 3, Chooser 2 gets piece 2, Chooser 3 gets piece 4.**
   - **Chooser 1 gets piece 4, Chooser 2 gets piece 1, Chooser 3 gets piece 2.**

   (c) (3 points) We are trying to divide 2 cups of water and 2 cups of sugar. Alfred likes sugar four times as much as much as water. If he was to get 1 cup of sugar, what percentage of the total would he think that was?

   Let's say Alfred things a cup of water is worth \$1 and a cup of sugar is worth \$4, so the total value of 2 cups water and 2 cups sugar is \$10. In his opinion, one cup of sugar is worth \$4, so this is \$4/\$10 = 40% of the total.

   (d) (5 points) We are trying to divide 3 square feet of granite tile and 6 square feet of sandstone tile between three contractors (A,B,C) using the last-diminisher method. Suppose in the first round, contractor A chooses 6 square feet of sandstone tile, and then contractor B has their turn.

   If contactor B likes granite three times as much as sandstone, what do they do?

   Again, let's assign monetary values. Let's say contractor B thinks that a square foot of sandstone is worth \$1 and a square foot of granite is worth \$3. In their opinion, then, the total value is $(3 \times \$3) + (6 \times \$1) = \$15$.

   If they got 6 square feet of sandstone, they think it is worth \$6, which is \$6/\$15 = 40% of the total, which is more than a fair share. They would therefore diminish it to 1/3 of the total (a fair share for them) and bid on it.

   In their opinion, a fair share is worth \$15/3 = \$5, and \$5 will buy 5 square feet of sandstone tile.

   So, in total: This player would diminish the sandstone to 5 square feet and then bid on it. (Whew!)

(e) (2 points) We are using the method of sealed bids to divide up a bell, a book, and a candle among three people (A,B,C). This is the result of their bidding:

	Bell	Book	Candle
A	\$9	\$4	\$5
B	\$10	\$5	\$3
C	\$3	\$3	\$3

Who gets which items?

The highest bidder for each item gets it, so B gets the bell and book, while A gets the candle. C gets nothing (yet).

(f) (4 points) How much money does each player lose or gain when we use the method of sealed bids to divide up the items?

The first step is that we figure out how much is a fair share for each player.

- **A bid a total of \$18, so a fair share for A is \$18/3 = \$6.**
- **B bid a total of \$18, so a fair share for A is \$18/3 = \$6.**
- **C bid a total of \$9, so a fair share for A is \$9/3 = \$3.**

Then we figure out how much they are short or got as extra.

- **A got \$5 worth of items and their fair share is \$6, so they are entitled to \$1 more.**
- **B got \$15 worth of items and their fair share is \$6, so they have to pay \$9.**
- **C got \$0 worth of items and their fair share is \$3, so they are entitled to \$3 more.**

This leaves \$9 - \$1 - \$3 = \$5 left over, which we have to divide evenly between the three players. (Oops, we can't really. But let's pretend.) So we give each of then \$5/3 = \$1$\frac{2}{3}$ back, which is about \$1.67.

So the final answer is:

- **In total, A gained \$1 + \$1$\frac{2}{3}$ = \$2$\frac{2}{3}$, or about \$2.67.**
- **In total, B lost \$9 - \$1$\frac{2}{3}$ = \$7$\frac{1}{3}$, or about \$7.33.**
- **In total, C gained \$3 + \$1$\frac{2}{3}$ = \$4$\frac{2}{3}$, or about \$4.67.**