Math 1001 Test 3 Solutions

1. True/False (20 points, 2 each). No partial credit.

   (a) **(TRUE)** A fractal is an object that is self-similar at different levels of magnification.

   (b) **(TRUE)** The Sierpinski gasket has zero area.

   (c) **(FALSE)** Playing the "chaos game" about 5 times in a row gives an interesting picture.

   (d) **(TRUE)** There are shapes in nature that look very similar to fractals.

   (e) **(FALSE)** The Mandelbrot set is formed by starting with a basic shape and applying some construction repeatedly to it.

   (f) **(FALSE, usually)** Multiplying the numbers 10 and 23 together is an example of a random experiment.

   (g) **(FALSE)** The probability of an event is the same as the probability of any outcome which is part of the event.

   (h) **(FALSE)** If the odds of winning are 2 to 1, that means that you have a 50% chance of winning.

   (i) **(TRUE)** There are 20 different ways to choose 3 CDs out of a stack of 6 if order doesn't matter.

   (j) **(TRUE)** It is possible to shuffle a deck of cards perfectly 8 times in a row and end up with them back in the order they started in.

2. Multiple choice (7 points). No partial credit.

   (a) (2 points) Which of the following shapes can be seen by playing the "chaos game"?

A	Sierpinski gasket	B	Koch snowflake
C	Mandelbrot set	D	Cantor set

   The correct answer is A.

   (b) (2 points) You are on a game show with 3 boxes, one with a prize hidden inside and two without, at random. If you guess that the prize is inside box 1, what is your chance of being correct?

A	1/3	B	1/2
C	2/3	D	None of the above

   The correct answer is A.

   (c) (3 points) In the same situation, after you chose the host accidentally reveals that the prize is not in box 2. Now what is the chance that you picked the correct box?

A	1/3	B	1/2
C	2/3	D	None of the above

   The correct answer is still A. Even though there are only two possibilities to choose from, you chose at random out of 3 and were really unlikely to have gotten the correct answer.

3. (12 points, 3 points each) Add up the following infinite geometric series. State your answer as a fraction, not in decimal form.

    All of these are gotten by using the formula $\frac{a}{1-r}$, where a is the first term in the series and r is the ratio between each term and the next.

    (a)
    $$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots = \frac{1}{1 - \frac{1}{2}} = 2$$

    (b)
    $$3 + \frac{3}{4} + \frac{3}{16} + \frac{3}{64} + \frac{3}{256} + \cdots = \frac{3}{1 - \frac{1}{4}} = 4$$

    (c)
    $$3 + 2 + \frac{4}{3} + \frac{8}{9} + \frac{16}{27} + \cdots = \frac{3}{1 - \frac{2}{3}} = 9$$

    (d)
    $$1 + 0.01 + 0.0001 + 0.000001 + \cdots = \frac{1}{1 - .01} = \frac{1}{99/100} = \frac{100}{99}$$

4. (15 points, 3 points each) Calculate the following using arithmetic of complex numbers.

    Note! The first two questions were about addition and subtraction, not multiplication, so we don't use the FOIL rule for those!

    (a)
    $$(3 - 4i) + (2 + 3i) = (3 + 2) + (-4i + 3i) = 5 - i$$

    (b)
    $$(1 - 5i) - (2 + 3i) = (1 - 2) + (-5 - 3i) = -1 - 8i$$

    (c)
    $$(1 + i) \times (1 + i) = 1 + i + i + i^2 = 2i$$

    (Remember $i^2 = -1$.)

    (d)
    $$(1 - 4i) \times (2 + 3i) = 2 + 3i - 8i - 12i^2 = 14 - 5i$$

    (e)
    $$(a + bi) \times (a - bi) = a^2 - abi + abi - b^2 i^2 = a^2 + b^2$$

5. (10 points, 2 points each) Independent events.

    We roll a die 3 times in a row. For each pair of events listed below, decide whether the two events are *independent* or not. If they are, mark with a Y; otherwise, mark with an N.

    Remember that events are independent only when knowing the results of one event tells you *nothing* about the results of the other.

Y/N	Event 1	Event 2
Y	First roll is 2 or 3	Second roll is a 3
N	Rolling three 1s	Second roll is a 5
Y	Third roll is a 2	First two rolls add up to 5
N	All three rolls add up to 4	First two rolls add up to 10
N	None of the rolls are 6	None of the rolls are 1

6. (16 points) We have an opaque jar filled with 100 marbles – 10 red, 30 blue, and 60 green – that are thoroughly mixed. Our random experiment is: We reach in and pull out two marbles, keeping track of which came out first and which came out second.

    (a) (3 points) Write down the sample space for this experiment.
        The sample space is $\{RR, RG, RB, GR, GG, GB, BR, BG, BB\}$.

    (b) (3 points) What is the probability of getting a red marble followed by a green marble?
        The number of ways to pick two marbles out of 100, with order mattering, is $_{100}P_2 = 100 \times 99 = 9900$. The number of ways to pick a red marble followed by a green marble is $10 \times 60 = 600$. So the probability is $\frac{600}{9900}$, or about 6%.

    (c) (2 points) Express the outcome "neither marble is blue" as a subset of the sample space.
        This outcome consists of everything in the sample space with neither marble being blue: the answer is $\{RR, RG, GR, GG\}$.

    (d) (4 points) What is the probability that neither marble is blue?
        Again, there are 9900 possible ways to pick marbles. There are 70 non-blue marbles, so the number of ways to pick two non-blue marbles is $_{70}P_2 = 70 \times 69 = 4830$. The probability is $\frac{4830}{9900}$, or about 48.7%.

    (e) (4 points) What is the probability that one of the marbles is green?
        The same way as in the last question, the probability that *neither* marble is green is $\frac{_{40}P_2}{_{100}P_2} = \frac{1560}{9900}$, or about 15.8%. Therefore, the probability that *at least one* of the marbles is green is the opposite, $100\% - 15.8\% = 84.2\%$. (If you assumed that the question was asking for *exactly* one of the marbles green, that was fine - I should have worded it more precisely.)