Math 2374	**Name (Print):** ______________
Spring 2007	**Student ID:** ______________
Final	**Section Number:** ______
May 7, 2007	**Teaching Assistant:** ______________
Time Limit: 3 hours	**Signature:** ______________

This exam contains 13 pages (including this cover page) and 12 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated. You are allowed to take one (doubled-sided) 8.5 inch $\times$ 11 inch sheet of notes into the exam.

Do not give numerical approximations to quantities such as $\sin 5$, π, or $\sqrt{2}$. However, you should simplify $\cos \frac{\pi}{4} = \sqrt{2}/2$, $e^0 = 1$, and so on.

The following rules apply:

- **Show your work**, in a reasonably neat and coherent way, in the space provided. **All answers must be justified by valid mathematical reasoning, including the evaluation of definite and indefinite integrals**. To receive full credit on a problem, you must show enough work so that your solution can be followed by someone without a calculator.
- **Mysterious or unsupported answers will not receive full credit**. Your work should be mathematically correct and carefully and legibly written.
- **A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit**; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- Full credit will be given only for work that is presented neatly and logically; work scattered all over the page without a clear ordering will receive very little credit.

1	30 pts	
2	25 pts	
3	25 pts	
4	25 pts	
5	15 pts	
6	30 pts	
7	20 pts	
8	30 pts	
9	25 pts	
10	25 pts	
11	25 pts	
12	25 pts	
TOTAL	300 pts	

1. (30 points) Let $f(x,y) = \frac{x^3}{3} + \frac{y^3}{3} - 2x^2 - y^2$. Find all of the critical points of $f(x,y)$, and classify each critical point as a local maximum, local minimum, or saddle point.

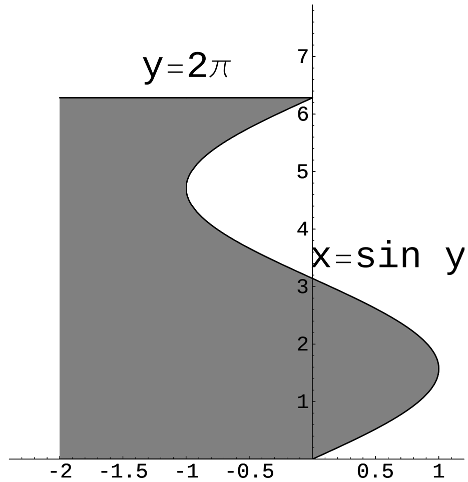
2. (25 points) Let $\mathbf{F}(x, y) = (ye^{xy} + 2x, xe^{xy} - \cos y)$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{s}$ where C is any curve from (1,0) to $(0, \pi/2)$.

3. (25 points) A population of bacteria is living on a plate. The density of bacteria at the point (x, y) is given by the function $f(x, y) = e^{1-x^2-2y^2}$.

   (i) (15 points) At the point $(x, y) = (1, 0)$, at what rate does the density increase in the direction $(-1, 1)$. (In other words, what is the slope of f in that direction?)

   (ii) (10 points) At the point $(x, y) = (1, 0)$, in what direction does the density increase most rapidly?

4. (25 points) Let D be the region in the xy-plane defined by $0 \le y \le 2\pi$ and $-2 \le x \le \sin y$ as shown below.



Let ∂D be the counterclockwise oriented boundary of D. Compute $\int_{\partial D} \mathbf{F} \cdot d\mathbf{s}$, where $\mathbf{F}(x, y) = (e^{x^2}, \sin(y^2) - x^2)$.

5. (15 points) Find an equation for the plane containing the line parametrized by $(x, y, z) = \mathbf{c}_1(t) = (1 + 2t, 2 - 3t, 3)$ and the line parametrized by $(x, y, z) = \mathbf{c}_2(t) = (-t, t + 3, 1 - 2t)$. Write your answer in the form $Ax + By + Cz + D = 0$.

6. (30 points) Let S be the surface which is the boundary of the cylindrical solid given by $x^2+y^2 \leq 9$ and $0 \leq z \leq 4$, with an outward pointing normal vector. Let $\mathbf{F}$ be the vector field given by

$$\mathbf{F} = \big(x, y, z^2(x^2 + y^2)\big)$$

Set up and evaluate a triple integral which will give you the flux of $\mathbf{F}$ across S. You must use a method that would "always work," i.e. making up a triple integral which just happens to have the correct answer will not result in any credit.

7. (20 points) Consider the surface defined by $z = f(x,y) = \sin(\pi xy) + x^2y - y^2 + 3$. Find the equation for the tangent plane at the point $(x,y,z) = (2,1,6)$.

8. (30 points) Let S be the paraboloid $z = (x^2 + y^2)/4$ for $z <= 4$ oriented with upward normal vector. Use Stokes' Theorem to calculate $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$, where

$$\mathbf{F}(x, y, z) = xy^2z\mathbf{i} - 4x^2y\mathbf{j} + \frac{z-1}{x^2 + 2y^2 + 1}\mathbf{k}.$$

9. (25 points) Let $\mathbf{r}(t) = \begin{bmatrix} \sin(2\pi t) \\ \cos(\pi t) \end{bmatrix}$ and $g : \mathbf{R}^2 \to \mathbf{R}$ be a function where $g(0,0) = 3$ and $\nabla g(0,0) = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$. Find $F'(1/2)$ where $F(t) = g(\mathbf{r}(t))$.

10. (25 points) Find the value of the line integral $\int_C y dx - x dy$ along the quarter unit circle C from the point $(0, 1)$ to the point $(1, 0)$.

11. (25 points) Consider the surface defined by $z = f(x, y) = 2x^2 + xy + y^2 - 3$. Find the quadratic approximation of the surface (i.e., second-order Taylor polynomial of f) at the point $(x, y, z) = (1, 2, 5)$. Use your approximation to estimate the value of $f(0.8, 2.1)$.

12. (25 points) Let S be the surface $z = x^2 + y^2 - 3$ for $-2 \leq z \leq 1$. Find the surface area of S.