Math 2374
Practice final exam answers and hints
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1. Four critical points: $(0,0)$ max ($D = 8$ and $f_{xx} = -4$), $(0,2)$ saddle ($D = -8$), $(4,0)$ saddle ($D = -8$), and $(4,2)$ min ($D = 8$ and $f_{xx} = 4$).

2. -2. (A potential function for $\mathbf{F}$ is $f = e^{xy} + x^2 - \sin y$.)

3. (a) $\sqrt{2}$.
   (b) $(-2, 0)$.

4. 7π. (Use Green's Theorem.)

5. $6x + 4y - z - 11 = 0$. (Use t-coefficients to get two vectors in the plane; use cross product to find normal.)

6. $$\int_0^{2\pi} \int_0^4 \int_0^3 (2 + 2zr^2) r \, dr \, dz \, d\theta = 720\pi.$$
   (Use Gauss' Theorem and cylindrical coordinates.)

7. $(\pi + 4, 2\pi + 2, -1) \cdot (x - 2, y - 1, z - 6) = 0$. (Use $(f_x, f_y, -1)$ to find normal.)

8. 0. The line integral resulting from Stokes' Theorem is
   $$\int_0^{2\pi} -4^5 \cos\theta \sin\theta \, d\theta.$$

9. 3π. (Use the chain rule. Note: ∇g is a 1×2 matrix, not a 2×1. Also note: $g(0,0) = 3$ is unnecessary information.)

10. $\frac{\pi}{2}$. (Use parametrization $\mathbf{c}(t) = (\sin t, \cos t)$, $0 \le t \le \frac{\pi}{2}$.)

11. $f(x,y) \approx 5 + 6(x-1) + 5(y-2) + 2(x-1)^2 + (y-2)^2 + (x-1)(y-2)$. $f(0.8, 2.1) \approx 4.37$. (It turns out to be the exact value, as well. Indeed, f itself is a second-degree polynomial.)

12. $\frac{\pi}{6}(17^{3/2} - 5^{3/2})$. (Use parametrization $\Phi(r,\theta) = (r\cos\theta, r\sin\theta, r^2 - 3)$ for $1 \le r \le 2$ and $0 \le \theta \le 2\pi$.)