Math 2374 **Name (Print):** ____________
Spring 2008 **Student ID:** ____________
Exam 1 **Section Number:** ______
February 27, 2008 **Teaching Assistant:** ____________
Time Limit: 1 hour **Signature:** ____________

This exam contains 6 numbered problems on four sheets of paper. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated. You are allowed to use one-half of one (doubled-sided) 8.5 inch $\times$ 11 inch sheet of notes.

Do not give numerical approximations to quantities such as $\sin 5$, π, or $\sqrt{2}$. However, you should simplify $\cos\frac{\pi}{4} = \sqrt{2}/2$, $e^0 = 1$, and so on.

Show your work, in a reasonably neat and coherent way, in the space provided. **All answers must be justified by valid mathematical reasoning.** To receive full credit on a problem, you must show enough work so that your solution can be followed by someone without a calculator. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.

1	20 pts	
2	20 pts	
3	20 pts	
4	25 pts	
5	25 pts	
6	30 pts	
TOTAL	140 pts	

1. (20 points) For each statement, either give an example that shows the statement is TRUE, or give a brief reason that the statement is FALSE.

   (a) (10) There exists a pair of nonzero vectors $\mathbf{v}$ and $\mathbf{w}$ in $\mathbf{R}^3$ such that $\mathbf{v} \times \mathbf{w} = \mathbf{0}$ and $\mathbf{v} \cdot \mathbf{w} = 0$.

   (b) (10) There exists a nonzero vector $\mathbf{v}$ in $\mathbf{R}^3$ such that $\mathbf{v} \cdot \mathbf{v} = ||\mathbf{v}||$.

2. (20 points) Suppose that $\mathbf{f}, \mathbf{g} : \mathbf{R}^2 \to \mathbf{R}^2$ are functions such that

$$\mathbf{f}(u, v) = (uv, u^2 - v^2)$$

and $\mathbf{g}(x, y) = (u(x, y), v(x, y))$ satisfies

$$\mathbf{g}(1, 2) = (5, -5)$$

and

$$D\mathbf{g}(1, 2) = \begin{bmatrix} 2 & 4 \\ 6 & -8 \end{bmatrix}.$$

(a) (10) Find $\frac{\partial v}{\partial x}(1, 2)$.

(b) (10) Find $D(\mathbf{f} \circ \mathbf{g})(1, 2)$.

3. (20 points) Let $z = f(x, y) = e^{-x}\sqrt{y}$.

   (a) (10) Find a function that is a linear approximation to $f(x, y)$ near the point $(0, 9)$. Write your linear approximation in the form $z = Ax + By + C$.

   (b) (10) A calculator will tell you that $e^{-.01}\sqrt{9.6} \approx 3.067557214$. Estimate this quantity using the linear approximation in (a), and write your answer to the hundredths place. Show your work.

4. (25 points) Let $F(x, y, z) = x^2y + yz + z^2$.

   (a) (15) Find $\nabla F(x, y, z)$.

   (b) (10) Are there any points (x, y, z) in upper-half-space (that is, where $z > 0$) at which the tangent plane to the level surface of F at that point is horizontal? Either find all such points, or give a reason why there are no such points.

5. (25 points) Let $g(x, y) = x + y^2$.

(a) (15) Find an equation for the plane that passes through the point $(3, -2, -4)$ and is parallel to the tangent plane to the graph of g at the point $(3, -2, 7)$. Write the equation in the form $Ax + By + Cz + D = 0$.

(b) (10) Sketch the level curve defined by the equation $g(x, y) = 5$ in the xy-plane. *Label the coordinate axes of your sketch.*

6. (30 points) Let $f(x, y) = 10 - x^2 - 3y^2$.

(a) (10) Write a unit vector $\mathbf{u}$ that indicates the direction in which f is decreasing the fastest at the point $(2, 1)$.

(b) (10) Find the directional derivative of f at the point $(2, 1)$ in the direction $\mathbf{u}$ that you found in part (a).

(c) (10) For each unit vector $\mathbf{v}$, let $\theta_{\mathbf{v}}$ denote the angle $0 \leq \theta_{\mathbf{v}} \leq \pi$ between $\mathbf{v}$ and the unit vector $\mathbf{u}$ that you found in (a). For which angles $\theta_{\mathbf{v}}$ is the directional derivative of f in the direction $\mathbf{v}$ at $(2, 1)$ less than or equal to -5? Write your answer as an inequality in the form $A \leq \theta_{\mathbf{v}} \leq B$.