Math 2374	**Name (Print):** ________________
Spring 2010	**Student ID:** ________________
Midterm 1	**Section Number:** ______
February 18, 2010	**Teaching Assistant:** ________________
Time Limit: 50 minutes	**Signature:** ________________

This exam contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated. You are allowed to take one-half of one (doubled-sided) 8.5 inch $\times$ 11 inch sheet of notes into the exam.

Do not give numerical approximations to quantities such as $\sin 5$, π, or $\sqrt{2}$. However, you should simplify $\cos \frac{\pi}{4} = \sqrt{2}/2$, $e^0 = 1$, and so on.

The following rules apply:

- **Show your work**, in a reasonably neat and coherent way, in the space provided. **All answers must be justified by valid mathematical reasoning, including the evaluation of definite and indefinite integrals**. To receive full credit on a problem, you must show enough work so that your solution can be followed by someone without a calculator.
- **Mysterious or unsupported answers will not receive full credit**. Your work should be mathematically correct and carefully and legibly written.
- **A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit**; an incorrect answer supported by substantially correct calculations and explanations will receive partial credit.
- Full credit will be given only for work that is presented neatly and logically; work scattered all over the page without a clear ordering will receive from little to no credit.

1	25 pts	
2	25 pts	
3	20 pts	
4	25 pts	
5	20 pts	
6	25 pts	
TOTAL	140 pts	

1. (25 points) Consider the triangle with vertices $(1, 0, 1), (0, 2, 0)$ and $(1, 2, 3)$.

   (a) (15 points) Calculate its area.

   (b) (10 points) Check if it is a right triangle.

2. (25 points) Consider the two lines given in parametric form by

$$\mathbf{c}_1(t) = (t+1, t+2, 2t-2) \qquad \text{and} \qquad \mathbf{c}_2(t) = (2t, 3, 1-t).$$

(a) (15 points) Suppose both lines $\mathbf{c}_1(t)$ and $\mathbf{c}_2(t)$ lie in the same level surface $f(x, y, z) = C$ of a differentiable function f. Find the tangent plane equation to the level surface at the point $(2, 3, 0)$.

(b) (10 points) If $f_x(2, 3, 0) = 2$, find the partial derivatives $f_y(2, 3, 0)$ and $f_z(2, 3, 0)$.

3. (20 points) Suppose the directional derivatives of function $f(x, y)$ at the point $(0, 0)$ in the directions $\vec{v}_1 = (1, 1)$ and $\vec{v}_2 = (2, 1)$ are given by

$$D_{\vec{v}_1} f(0, 0) = 1$$

and

$$D_{\vec{v}_2} f(0, 0) = 2,$$

respectively. Find the gradient $\nabla f(0, 0)$.

4. (25 points) Consider the function $g : \mathbb{R}^3 \to \mathbb{R}^2$ given by the expression

$$g(x, y, z) = (x - y + 3z,\ 2x + z^2).$$

Suppose for function $f : \mathbb{R}^2 \to \mathbb{R}^2$, $f(x, y) = (f_1(x, y), f_2(x, y))$, we have

$$\mathbf{D}f(8, 9) = \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix}$$

Find $\nabla(f_2 \circ g)(0, 1, 3)$.

5. (20 points) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given by

$$f(x, y) = 3xy - 2y^2 - 4x.$$

Find all the points on the graph of f at which the tangent plane to the graph of f is parallel to the plane given by

$$x + 4y + z = 1.$$

6. (25 points) A hot air balloon is positioned at a point with coordinates $(1,1,1)$. The pressure of air as a function of the space coordinates is given by

$$P(x,y,z) = \frac{x^2 + (y-1)^2}{1+z^2}.$$

(a) (15 points) If the balloon goes in the direction of largest decrease of pressure, what would that direction be? (Give your answer as a unit vector)

(b) (10 points) If the balloon takes the direction computed in part (a) and has a speed of 5 units of space per unit of time, what would be the rate of change of pressure felt by the crew?