Math 5286H

Problem Set 1

Due on **Monday, February 6.**

True/false. Correct answers are 2 points, incorrect worth 0 points, "I don't know" worth 1 point.

________ If I and J are ideals of a ring R, then $I \cup J$ must be an ideal.

________ If R is a commutative ring and $x \in R$, then the set $\{r'xr \mid r', r \in R\}$ is an ideal.

________ If R is a ring, the set of 2×2 upper-triangular matrices

$$\left\{ \begin{bmatrix} a & b \\ 0 & d \end{bmatrix} \middle| a, b, d \in R \right\}$$

is a subring of $M_2(R)$.

________ For any ring R, the set $\{r \in R \mid r \neq 0\}$ is a group under the operation of multiplication.

________ In a Euclidean domain, every ideal is a principal ideal.

Short answer. 5 points each for a correct answer.

1. If n and m are integers, then $(n) \cap (m)$ of $\mathbb{Z}$ is the ideal generated by the ________ of n and m.
2. In the ring $\mathbb{R}[x]$, the intersection of the ideals $(x^5 + x^3 - 2)$ and $(x^4 - 3x^3 + 2)$ is the ideal generated by the monic polynomial ________.

Long form. 10 points.

1. The ring $\mathbb{Z}[x, y]/(x + 6y + 7, x - y, x + y + 1)$ is isomorphic to $\mathbb{Z}/n$ for some positive n. Give a proof (including finding the correct value of n).