Math 5345H, Fall 2015
Midterm 1
Due in-class on **Wednesday, October 21**

All questions have equal value.

1. List all the possible topologies on the set with 3 points, up to homeomorphism. For each, identify whether it is
   (a) T_0,
   (b) T_1,
   (c) T_2,
   (d) connected.
2. Identify the values of n for which the following statement is true: Suppose X is a space and $A_1, A_2, \ldots, A_n \subseteq X$ are connected subspaces such that $\bigcup A_i = X$ and $A_i \cap A_j \neq \emptyset$ for all i, j. Then X is connected.
3. Give an example of a metric space X with a closed, bounded subset $K \subseteq X$ which is not compact.
4. Let X and Y be topological spaces and $f : X \to Y$ a function. Define the *graph* of f to be
$$\Gamma = \{(x, y) \in X \times Y \mid y = f(x)\}.$$
   Consider the following statement: If Γ is a closed subset of $X \times Y$ then f is continuous.
   Prove this statement or provide a counterexample. Then do the same for the converse.
5. Prove that the circle and square
$$\{(x, y) \in \mathbb{R}^2 \mid 1 = x^2 + y^2\} \qquad \{(x, y) \in \mathbb{R}^2 \mid 1 = |x| + |y|\}$$
   are homeomorphic.
6. Prove that the open and closed unit balls
$$\{(x, y) \in \mathbb{R}^2 \mid 1 \geq x^2 + y^2\} \qquad \{(x, y) \in \mathbb{R}^2 \mid 1 > x^2 + y^2\}$$
   are not homeomorphic.