

Math 5378, Differential Geometry  
Homework 3  
Due in-class on **Wednesday, February 13**

Numbered exercises are from Do Carmo, *Differential Geometry of Curves and Surfaces*.

1. Section 1.5, Exercise 2.
2. Section 1.5, Exercise 6.
3. Section 1.5, Exercise 12.
4. Section 2.1, Exercise 1.
5. Section 2.1, Exercise 2.
6. Section 2.1, Exercise 7.
7. Using the inverse function theorem, find all points  $(x, y, z) \in \mathbb{R}^3$  such that the function

$$f(x, y, z) = (x^2 + y^2, z^2 + z, xy)$$

has a differentiable inverse in a neighborhood of  $(x, y, z)$ .

8. We define a function

$$f(x, y, z) = x^2 + y^2 + z^2 + xy + xz + yz - x - y - z$$

Show that the equation  $f(x, y, z) = 0$  defines a smooth surface in  $\mathbb{R}^3$ .