Math 8301, Manifolds and Topology
Homework 1
Due in-class on **Friday, Sep 14**

In the following questions, $\#$ denotes the connected sum of surfaces.

1. Show that any open subset of a manifold is a manifold.

2. For each value of $t \in \mathbb{R}$, decide whether the space

$$\{(x, y, z) \in \mathbb{R}^3 \mid xyz = t\}$$

   is a manifold, and explain why or why not.

3. For which values of $t \in \mathbb{R}$ is the space

$$\{(x, y) \in \mathbb{R}^2 \mid x^2 + xy + ty^2 = 1\}$$

   a *closed* manifold?

4. If M is any surface and S^2 is the 2-sphere, explain why $S^2 \# M \cong M$.

5. Show that, if P is a copy the real projective plane, $P \# P$ is homeomorphic to a Klein bottle.