Math 8301, Manifolds and Topology
Homework 2
Due in-class on **Friday, Sep 21**

1. Show graphically that the simplicial complex with 7 vertices, generated by the triangles below, gives rise to a space homeomorphic to the torus.

123	127	134	145	156	167	236
245	246	257	347	356	357	467

2. For a 2-dimensional simplicial complex with v vertices, e edges, and f triangles, the *Euler characteristic* χ is defined to be $v - e + f$. If this simplicial complex gives rise to a compact surface, give formulas for e and f in terms of χ and v which are nondecreasing in v.

3. Using the formulas from the previous problem, show that any triangulation of a compact surface of Euler characteristic 0 requires at least 7 vertices, and any of Euler characteristic 1 requires at least 6 vertices.

4. By identifying points on opposite sides of an icosahedron, give a triangulation of $\mathbb{RP}^2$ with 6 vertices and 10 faces.

5. Suppose that you are given a simplicial complex with set $\mathcal{V}$ of vertices and set $\mathcal{F} \subset \mathcal{P}(\mathcal{V})$ of faces, satisfying two properties.

   (a) Any face $U \in \mathcal{F}$ satisfies $|U| \leq 3$.

   (b) For any vertices $a \neq b$ such that $\{a, b\} \in \mathcal{F}$, there are precisely two *other* vertices c such that the three-element set $\{a, b, c\}$ is in $\mathcal{F}$.

   Does this simplicial complex necessarily give rise to a compact surface? Either give a proof or a counterexample. (Be careful.)