Math 8301, Manifolds and Topology

Homework 8

Due in-class on **Friday, November 9**

1. Show that S^2 is isomorphic to the universal covering space of $\mathbb{RP}^2$.

2. Give a description of the universal cover of the space $S^2 \vee S^1$, obtained by gluing together S^2 and S^1 at a single point.

3. Suppose X and Y are path-connected spaces, $p : Y \to X$ is a covering map, and $y \in Y$. Let the image of $\pi_1(Y, y)$ in the fundamental group $G = \pi_1(X, p(y))$ be the subgroup H. Let NH be the normalizer of H in G. Show that there is a bijection between *deck transformations* (*homeomorphisms* $f : Y \to Y$ with $pf = p$) and elements of NH/H which takes function composition to group multiplication.

4. Using the previous homework, show that the fundamental group of a *connected graph* with v vertices and e edges (both finite) is always a free group, with number of generators equal to $1 - v + e$.

5. Suppose F is a free group with n generators. Show there is a graph with fundamental group F. If $H < F$ is a subgroup with $[F : H] = m < \infty$, use the previous exercise and covering space theory to show that H is a free group with $1 + m(n - 1)$ generators.