Math 8302, Manifolds and Topology II
Auxiliary problems
Due in-class on **Monday, April 29**

The interaction between these problems and the final is as stated in class.

1. Suppose M is a smooth manifold of dimension n. A *smoothly time-dependent* 1-form on M is a family of 1-forms $\omega(t)$ for $t \in (a,b)$ such that, in any coordinate chart $(x^1, \ldots, x^n)$, the coordinate expression of ω is of the form
$$\sum_{k=1}^{n} f_k(x^1, \ldots, x^n, t)\, dx^k$$
where each f_k is a smooth function of $(x^1, \ldots, x^k, t)$.

   Give (and prove) the most concise condition that you can find for ω to satisfy the the following.

   For any time interval $[c,d] \subset (a,b)$ and any two smooth curves $\gamma_1, \gamma_2 : [c,d] \to M$, the line integrals of $\omega(t)$ over γ_1 and γ_2 agree.

   Note that such a line integral involves the change of $\gamma_i(t)$ and substituting the time t into the 1-form $\omega(t)$. For example, such a line integral would be calculated in coordinates as
$$\sum_{k=1}^{n} \int_c^d f_k(x^1(t), \ldots, x^n(t), t)\, \frac{dx^k}{dt}$$

2. For $n > 0$, the manifold $\mathbb{CP}^n$ contains $\mathbb{CP}^1$ as a submanifold, and the second de Rham cohomology group $H^2_{dR}(\mathbb{CP}^n)$ is $\mathbb{R}$. Find, for each n, a generator of H^2_{dR} in the following way: give an explicit *closed* 2-form, take the integral over $\mathbb{CP}^1$, and show that this integral is nonzero.

   Do the same for the complex Grassmannian $Gr_{\mathbb{C}}(2,4)$.

3. Track down the definition of the Moore manifold from Royden's real analysis text. Prove that it is a manifold. (Unless you're reading someone's transcription, there is probably not a mistake in the problem.)