Math 8306, Algebraic Topology
Homework 10
Due in-class on **Monday, November 10**

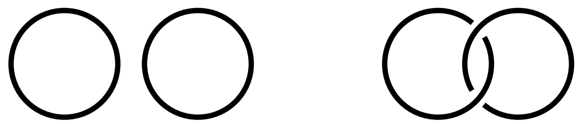

Figure 1: Two links in $\mathbb{R}^3$

1. The left-hand portion of the above picture is a union L_1 of two disconnected circles in $\mathbb{R}^3$. Show that the complement $X = \mathbb{R}^3 \setminus L_1$ retracts down onto $S^2 \vee S^2 \vee S^1 \vee S^1$. Use this to show that the cup product of any two elements in $H^1(X)$ is zero.

2. The right-hand portion of the picture is a link L_2 in $\mathbb{R}^3$. Show that the complement $Y = \mathbb{R}^3 \setminus L_2$ has the same cohomology as the space X from the previous problem. (Possible hint: Show that the space retracts down onto something gotten by gluing two torii together along $S^1 \vee S^1$. Don't appeal to any major duality theorems like Alexander duality.)

3. Show that the cup product of the two generators in $H^1(Y)$ is nonzero. (Possible hint: Compare it with a torus.)