Math 8306, Algebraic Topology
Homework 2
Due in-class on **Monday, September 15**

Numbered exercises are from Hatcher's "Algebraic Topology."

1. Suppose we have a Δ-set X with

$$X_0 = \{p\}$$
$$X_1 = \{a, b, c\}$$
$$X_2 = \{u, v\}$$

   and face maps

$$\begin{array}{ccc} \partial^i(a) = p & \partial^i(b) = p & \partial^i(c) = p \\ \partial^0(u) = a & \partial^1(u) = c & \partial^2(u) = b \\ \partial^0(v) = b & \partial^1(v) = c & \partial^2(v) = a \end{array}$$

   What is the resulting space?

2. Construct a Δ-set whose geometric realization is the 2-sphere S^2.

3. Compute the simplicial homology groups $H_n(\mathbb{RP}^2; \mathbb{Z})$ using the Δ-complex structure given in class.

4. Hatcher, exercise 4 on page 131.

5. Suppose $f : A \to B$ and $g : B \to C$ are homomorphisms of abelian groups. Show that there is an exact sequence

$$0 \to \ker(f) \to \ker(gf) \to \ker(g) \to \operatorname{coker}(f) \to \operatorname{coker}(gf) \to \operatorname{coker}(g) \to 0.$$