Math 8306, Algebraic Topology
Homework 4
Due in-class on **Monday, September 29**

Short homework this week due to the delay in posting.

1. (Formal Mayer-Vietoris sequence) Suppose that there is a map of long exact sequences as follows:

$$\begin{array}{ccccccccccccc} \cdots & \longrightarrow & F_{n+1} & \longrightarrow & A_n & \longrightarrow & B_n & \longrightarrow & F_n & \longrightarrow & A_{n-1} & \longrightarrow & \cdots \\ & & \downarrow{\scriptstyle\sim} & & \downarrow & & \downarrow & & \downarrow{\scriptstyle\sim} & & \downarrow & & \\ \cdots & \longrightarrow & G_{n+1} & \longrightarrow & C_n & \longrightarrow & D_n & \longrightarrow & G_n & \longrightarrow & C_{n-1} & \longrightarrow & \cdots \end{array}$$

   Here all the maps $F_n \to G_n$ are isomorphisms. Show that there is a long exact sequence:

$$\cdots \to D_{n+1} \to A_n \to B_n \oplus C_n \to D_n \to A_{n-1} \to \cdots$$

   (Define the maps first.)

2. Suppose X is a CW-complex with finitely many cells. Define the Euler characteristic $\chi(X)$ to be the number of even-dimensional cells minus the number of odd-dimensional cells.

   If F is *any* field, show that

$$\chi(X) = \sum_i (-1)^i \dim_{\mathbb{F}}(H_i(X;F))$$

   (Hint: Express this in terms of the ranks of the boundary maps.)