Math 8306, Algebraic Topology
Homework 6
Due in-class on **Monday, October 13**

1. Find a pair of chain complexes C_* and D_* such that the tensor product chain complex $C_* \otimes D_*$ does not satisfy the Kunneth formula, i.e. there is some n such that

$$H_n(C_* \otimes D_*) \neq \bigoplus_{p+q=n} H_p(C_*) \otimes H_q(D_*) \oplus \bigoplus_{p+q=n-1} \mathrm{Tor}(H_p(C_*), H_q(D_*)).$$

2. Suppose G is a topological group and X is a topological space with a continuous map $G \times X \to X$ which is an action of G. Show that $H_*(X)$ becomes a left module over the Pontrjagin ring $H_*(G)$.

3. Find the homology of the complex Grassmann $\mathrm{Gr}_{\mathbb{C}}(3,5)$.

4. There is a continuous map from one Grassmannian $\mathrm{Gr}(k,n)$ to the next $\mathrm{Gr}(k,n+1)$ by sending a plane $V \subset \mathbb{R}^n$ to the plane

$$\{(0, x_1, \ldots, x_n) | (x_1, \ldots, x_n) \in V\}.$$

   Show that the image consists of a union of Schubert cells, and find the dimension of the smallest cell not in the image.