Math 8307, Algebraic Topology II
Homework 1
Due in-class on **Wednesday, January 28**

1. Suppose A is a set with two binary operations $\circ$, $*$ that share a common two-sided unit e: for any $a \in A$,

$$a = e \circ a = a \circ e = e * a = a * e.$$

Suppose that these two operations additionally satisfy an interchange law

$$(a * b) \circ (c * d) = (a \circ c) * (b \circ d).$$

Show that this implies $a * b = a \circ b$ for any $a, b \in A$, and additionally $a \circ b = b \circ a$.

2. Suppose X is a space with a multiplication operation $* : X \times X \to X$ having unit $x_0 \in X$, and let $A = \pi_1(X, x_0)$. Show that path composition $\circ$ and pointwise multiplication $(f * g)(t) = f(t) * g(t)$ satisfy the conditions of the previous problem, and hence the multiplication on $\pi_1(X, x_0)$ is commutative.

3. Let $B = [0,1] \times [0,1]$ be the unit square with boundary ∂B. Suppose X is a space with a chosen basepoint x_0, and let A be the set of maps $f : B \to X$ such that $f(\partial B) = \{x_0\}$. By analogy with the fundamental group, define two "multiplication" operations $\circ, * : A \times A \to A$ that satisfy the interchange rule listed in problem 1. (They have no unit until we pass to homotopy classes of maps.)

4. Hatcher, exercise 2 on page 358.