Math 8307, Algebraic Topology II
Homework 10
Due in-class on **Wednesday, April 15**

1. An alternate form of the Adem relations is given by

$$\sum_{j=0}^{k} \binom{k}{j} Sq^{2n-1-j} Sq^{n-k+j} = 0.$$

   This is valid for $0 \leq k \leq n$. Show that these equations for $k = 0, 1, 2, 3$ are equivalent to four of the Adem relations.

2. Show that the following elements commute with each other and square to zero, generating an exterior algebra on three generators.

   - $Q_0 = Sq^1$
   - $Q_1 = [Sq^2, Q_0] = Sq^2 Sq^1 + Sq^1 Sq^2$

   (These are referred to as the first two *Milnor primitives*. The Milnor primitives are defined (in one way) by $Q_{i+1} = [Sq^{2^i}, Q_i]$, and generate an exterior algebra on infinitely many generators.)

3. Describe $H^*(\mathbb{CP}^3 \times \mathbb{CP}^2)$ together with its action of the mod-2 Steenrod algebra.

4. Find the smallest possible subalgebra of the mod-2 Steenrod algebra generated by Sq^1 and Sq^2.