Math 8307, Algebraic Topology II
Homework 4
Due in-class on **Wednesday, February 18**

1. A map $f : X \to Y$ is called an *acyclic Serre fibration* if, whenever we have a commutative diagram

$$\begin{array}{ccc} S^n & \longrightarrow & X \\ \downarrow & & \downarrow \\ D^{n+1} & \longrightarrow & Y, \end{array}$$

   we can find a lift to a map $D^{n+1} \to X$ to make the diagram commute. Show that acyclic Serre fibrations are, in particular, Serre fibrations.

2. Show that an acyclic Serre fibration gives an isomorphism on homotopy groups.

3. Suppose X is a CW-complex whose cells are of dimension d or less and Y is a space with $\pi_n(Y) = 0$ for $n \leq d$. Show that any map $X \to Y$ is null-homotopic.

4. A connected space Y that has only one nonzero homotopy group,

$$\pi_d(Y, y) = \begin{cases} G & \text{if } d = n, \\ 0 & \text{otherwise,} \end{cases}$$

   is called an Eilenberg-Maclane space $K(G, n)$. Show that, for any CW-complex X, the set $[X, K(G, n)]$ only depends on the quotient $X^{(n+1)}/X^{(n-2)}$ of the $(n+1)$-skeleton by the $(n-2)$-skeleton.