Math 8307, Algebraic Topology II
Homework 6
Due in-class on **Wednesday, March 11**

Eilenberg-Maclane spaces.

1. Suppose G is an abelian group and $n \geq 2$. We know from algebra that we can find an exact sequence

$$0 \to R \to F \to G \to 0$$

   where F is free on some set of generators $\{e_\alpha | \alpha \in A\} \subset F$ for A and R is free on some set of relations $\{f_\beta | \beta \in B\} \subset R$. Show that we can find a map

$$\bigvee_{\beta \in B} S^n \to \bigvee_{\alpha \in A} S^n$$

   such that the induced map on π_n is isomorphic to the map $R \to F$.

2. (continuing the previous problem) Show that we can construct a CW-complex X, having cells only in dimensions n and $(n+1)$, with $\pi_k(X) = 0$ for $k < n$ and $\pi_n(X) = G$.

3. (still continuing) Show that we can construct a CW-complex $K(G, n)$, having cells only in dimensions n and higher, with the only nonzero homotopy group being $\pi_n(K(G, n)) = G$.

4. (still continuing) Given any based space Y with $\pi_n(Y) = G$ and $\pi_k(Y) = 0$ for $k > G$, show that we can construct a map $K(G, n) \to Y$ which is an isomorphism in dimension n.

   Then use the Whitehead theorem to conclude that any two CW complexes with the only nonzero homotopy group, π_n, being isomorphic to G are homotopy equivalent.