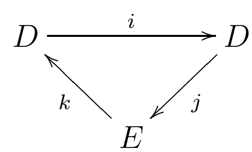
Math 8307, Algebraic Topology II
Homework 8
Due in-class on **Wednesday, April 1**

One way that spectral sequences arise is through what is called an *exact couple*. It arises algebraically in the following situation (which is a simplification of one we would actually use).

Suppose that we have abelian groups D and E, and maps $i : D \to D$, $j : D \to E$, and $k : E \to D$ such that the

$$\cdots \to D \to D \to E \to D \to D \to E \to \cdots$$

is exact. We often write this in a triangle:

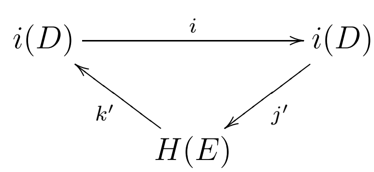


1. Show that the map $d = jk$ satisfies $d^2 = 0$, and so we get a homology group $H(E) = \ker(d)/\mathrm{Im}(d)$.

2. Let $i(D) = \mathrm{Im}(i)$. Show that the map k induces a well-defined map $k' : H(E) \to i(D)$.

3. Show that the description $j'(x) = ji^{-1}(x)$ gives us a well-defined map $i(D) \to H(E)$.

4. Show that these maps give us a new exact couple:



In other words, show that these three maps give a new long exact sequence. This is called the *derived* exact couple, and this procedure can be iterated again, and again, and again...