18.704 Problem Set 4

Due Tuesday, April 11, at **3pm** in 2-171

At least one of your answers must be typeset in TEX.

(You don't need to submit the TEX code.)

This week, you have a choice.

EITHER do questions 1 and 2, **OR** do question 3.

(Or do everything if you want extra credit.)

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1. List all the subgroups H of S_3 together with the characters of their irreducible representations. Use the formula for induced representations to find the characters of all the induced representations. Then use this to write all of these induced representations as direct sums of irreducible representations.

2. Suppose H is a subgroup of G and $\rho : H \to \mathrm{GL}(V)$ is a representation of H. Define

$$W = \{f : G \to V \mid f(hx) = \rho(h)f(x) \text{ for all } h \in H\}$$

We define

$$(\alpha f_1 + f_2)(x) = \alpha f_1(x) + f_2(x).$$

Show that if $f_1, f_2 \in W$, $\alpha \in \mathbb{C}$, then $\alpha f_1 + f_2 \in W$, so W is a vector space. We define $g \cdot f$ by

$$(g \cdot f)(x) = f(xg^{-1}).$$

Show that $g \cdot f \in W$, and that this gives a representation of G on W. (This gives an explicit construction of the induced representation of V.)

3. Compute the character table of the symmetric group S_5 using the method introduced in Li-Mei's talk. (There is a description of this method on the course website under the "notes" section.) Show your work!

* Pick a topic for your writing project! There is a list of topics on the course website.