18.906 Problem Set 10

Due Wednesday, April 25 in class

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1. Suppose that $\xi \to X$ is a 3-dimensional complex vector bundle. Use symmetric polynomials to express the Chern classes of $\bigwedge^2 \xi$ in terms of those of ξ. Same question for Stiefel-Whitney classes.

2. Also using symmetric polynomials, find explicit formulas for the Chern character in terms of Chern classes when $\xi \to X$ is a 2-dimensional vector bundle. Same question when X is restricted to only have nonzero cohomology in dimensions 0 through 4 but ξ can have arbitrary dimension.

3. The *octonions* $\mathbb{O}$ are an 8-dimensional non-associative division algebra over $\mathbb{R}$; there is a bilinear multiplication $\mathbb{O} \times \mathbb{O} \to \mathbb{O}$ such that $x \cdot y = 0$ if and only if $x = 0$ or $y = 0$. Let $\mathbb{O}^\times$ be the set of nonzero elements of $\mathbb{O}$

   Explain what goes wrong if we try to define octonion projective space via the formula
   $$\mathbb{OP}^n = \mathbb{O}^{n+1}/\{v \sim \alpha v, \alpha \in \mathbb{O}^\times\};$$
   in particular there is not a fiber bundle $\mathbb{O}^{n+1} \setminus \{0\} \to \mathbb{OP}^n$ with fiber $\mathbb{O}^\times$.

4. Compute $K(\mathbb{RP}^2)$ as a ring.