18.906 Problem Set 4 Alternate Question

Due Wednesday, March 7 in class

If you don't know anything about Lie groups, you can skip problem 2 on problem set 4 and prove the following instead.

Suppose G is a topological group, i.e., a topological space with a group structure such that the multiplication map $\mu : G \times G \to G$ and the inverse map $\nu : G \to G$ are continuous. Suppose G has a continuous action on a space X. Suppose that there exist a *transverse slices* to G: for any point $x \in X$ there is a subspace $U \subset X$ such that the map $G \times U \to X$ sending (g, u) to $g \cdot u$ is a homeomorphism onto an open subset.

Show that the projection map $X \to X/G$ is a fiber bundle with fiber G.

If $X/G = D^n$, show that there is a homeomorphism $f : G \times D^n \to X$ such that $f(gh, t) = g \cdot f(h, t)$ for any $g, h \in G, t \in D^n$.