

Math 1142
Spring 2003
FINAL EXAM

Name (Print) _____
Signature _____

Copy 1

Recitation Instructor _____ Section _____

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READ AND FOLLOW THESE INSTRUCTIONS

This booklet contains 16 pages, including this cover page and a blank page for scratch at the end. Check to see if any are missing. PRINT on the upper right-hand corner all the requested information, and sign your name. Put your initials on the top of every page, in case the pages become separated. Textbooks and notes are **not permissible**. Only scientific calculators are allowed (not graphing). Do your work in the blank spaces and back of pages of this booklet. Show all your work.

Every machine-graded problem counts 10 points. The 6 hand-graded problems are of varying points. The total score is 300 points.

INSTRUCTIONS FOR MACHINE-GRADED PART (Questions 1-9):

You MUST use a soft pencil (No. 1 or No 2) to answer this part. Do not fold or tear the answer sheet, and carefully enter all the requested information according to the instructions you receive. **DO NOT MAKE ANY STRAY MARKS ON THE ANSWER SHEET.** When you have decided on a correct answer to a given question, circle the answer in this booklet and blacken completely the corresponding circle in the answer sheet. If you erase something, do so completely. Each question has a correct answer. If you give two different answers, the question will be marked wrong. There is no penalty for guessing, but if you don't answer a question, skip the corresponding line in the answer sheet. Go on to the next question.

INSTRUCTIONS FOR THE HAND-GRADED PART (Questions 10-15):

SHOW ALL WORK. Unsupported answers will receive little credit.

Notice regarding the machine graded sections of this exam: Either the student or the School of Mathematics may for any reason request a regrade of the machine graded part. All regrades will be based on responses in the test booklet, and not on the machine graded response sheet. Any problem for which the answer is not indicated in the test booklet, or which has no relevant accompanying calculations will be marked wrong on the regrade. *Therefore work and answers must be clearly shown on the test booklet.*

AFTER YOU FINISH BOTH PARTS OF THE EXAM; Place the answer sheet between two pages of this booklet (make a sandwich), with the side marked "GENERAL PURPOSE ANSWER SHEET" facing **DOWN**. Have your ID card in your hand when turning in your exam.

Multiple choice part _____ Hand-graded part _____

Total _____

Letter Grade _____

Problem	
10	
11	
12	
13	
14	
15	
Subtotal	

Multiple Choice Problems for the 1142 Final

1. Which one of the following statements is true?

A) $\lim_{x \rightarrow 1} \frac{|x-1|}{x-1} = 0$

B) $\lim_{x \rightarrow 1} \frac{|x-1|}{x-1} = 1$

C) $\lim_{x \rightarrow 1} \frac{|x-1|}{x-1} = -1$

D) $\lim_{x \rightarrow 1} \frac{|x-1|}{x-1}$ does not exist

E) None of the statements above is true.

2. The equation of the tangent line to the graph of the function $y = \frac{1}{x} + x$ at the point

(1, 2) is:

A) $y = 2$

B) $y = 2x$

C) $y = \frac{1}{2}x + \frac{3}{2}$

D) $y = (\ln(2) + 2)x - \ln(2)$

E) $y - 2 = \left(-\frac{1}{x^2} + 1\right)(x - 1)$

3. The maximum and minimum values taken by the function $f(x) = 2x^3 - 3x^2 - 12x + 5$ on the interval $[-3, 3]$ are:

A) 2 and -1

B) 12 and -15

C) 12 and -40

D) -1 and -3

E) 17 and -15

4. Suppose that $f(x) = e^x \ln(x) + x^{\frac{1}{3}} - 2 - \frac{1}{x^2 + 1}$. Then, the derivative of $f(x)$ is:

A) $xe^x + e^x \ln(x) + \frac{1}{3}x^{\frac{2}{3}} + \frac{2x}{x^2 + 1}$

B) $\frac{1}{3}x^{\frac{2}{3}} + \frac{e^x}{x} + x \ln(x) + \frac{x}{(x^2 + 1)^2}$

C) $\frac{e^x}{x} + e^x \ln(x) + 3x^{\frac{2}{3}} + \frac{x}{2(x^2 + 1)^2}$

D) $\frac{e^x}{x} + e^x \ln(x) + \frac{1}{3}x^{-\frac{2}{3}} + \frac{2x}{(x^2 + 1)^2}$

E) $e^{x-1} + e^x \ln(x) + \frac{1}{3}x^{-\frac{2}{3}} + \frac{2x}{(x^2 + 1)^2}$

5. Let $f(x) = \frac{x}{x-1}$ be a function defined so long as $x \neq 1$. Then, $f''(x)$ is:

A) $x^2(x-1)^{-2}$

B) $x(x-1)^{-2}$

C) $2(x-1)^{-3}$

D) $x(x-1)$

E) $f''(x)$ does not exist

6. The value of $\int_1^3 \frac{\ln(x)}{x} dx$ is:

A) $\ln(3)$

B) $\frac{(\ln(3))^2}{2}$

C) $2 - \ln(3)$

D) $\frac{e^4}{3} + 1$

E) $-(\ln(3))^2$

7. The average value of $f(x) = (x-1)^3$ on the interval $[0, 2]$ is:

A) $-\frac{3}{4}$

B) 0

C) $\frac{3}{8}$

D) $\frac{1}{2}$

E) $\frac{2}{3}$

8. Which of the following statements is true?

A) $\int_0^{\infty} 5e^{-5x} dx$ does not converge.

B) $\int_0^{\infty} 5e^{-5x} dx = 1$

C) $\int_0^{\infty} 5e^{-5x} dx = 5$

D) $\int_0^{\infty} 5e^{-5x} dx = e^{-5}$

E) $\int_0^{\infty} 5e^{-5x} dx = -5$

9. If $f(x, y) = (x^2 + y^2)e^{xy}$, then $\frac{\partial^2 f}{\partial x \partial y} =$

A) $(3x^2 + 3y^2 + x^3y + xy^3)e^{xy}$

B) $(2 + 4xy + x^2y^2 + y^4)e^{xy}$

C) $(2 + 4xy + x^2y^2 + x^4)e^{xy}$

D) $2x^2e^{xy}$

E) $2ye^{xy}$

10. Consider the function $y = x^4 - 4x^3 + 12$.

- A) Find the critical points , (6 points)
- B) Which of the critical points are, respectively, local maxima, local minima, or inflection points? (8 points)
- C) Where is the function increasing? Where does it decrease? (8 points)
- D) Where is the graph concave up? Where is it concave down? (8 points)
- E) Sketch the curve for $-1 \leq x \leq 4$. (10 points)

11. The equation $x^2 - 3xy + y^3 = -1$ defines a curve in the plane.

A) At a point where this curve is the graph of a function y of x , use implicit differentiation to find $\frac{dy}{dx}$. (20 points)

B) Use the formula that you obtained in part A) to find the equation of the tangent line to this curve at the point $(2, 1)$. (15 points)

12. Find the area of the region bounded above by $y = xe^{-x^2}$, $x \geq 0$, and below by the line $y = 0$. (30 points)

13. A particle travels along the X-axis. Its position at time t , $0 \leq t \leq 10$, is given by $x(t) = 360,000 - e^{\frac{t}{10}}(360,000 - 36,000t + 1800t^2 - 60t^3 + t^4)$. Find the velocity of the particle at time t , $0 \leq t \leq 10$. Then show that the particle starts from the origin with velocity zero and travels to the right for all t , $0 < t \leq 10$. (35 points)

14. The function $f(x) = \frac{x^4}{625}$ is a probability density function on the interval $[0, 5]$.

A) Find the mean of this probability density function. (15 points)

B) Find the standard deviation of this probability density function.

(20 points)

15. The weights of the students in a certain college are normally distributed with mean 155 lbs. and standard deviation 35 lbs. What percentage of the student body is expected to have weights between 145 lbs. and 160 lbs.? You may use the shortened table of areas under the standard normal distribution curve given below.

t	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517

(35 points)