

Math 1142  
Fall 2005  
FINAL EXAM

Name (Print) \_\_\_\_\_

Signature \_\_\_\_\_

Recitation Instructor \_\_\_\_\_ Section \_\_\_\_\_ I.D.# \_\_\_\_\_

**READ AND FOLLOW THESE INSTRUCTIONS**

This booklet contains 12 pages, including this cover page. Check to see if any are missing. PRINT on the upper right-hand corner all the requested information, and sign your name. Put your initials on the top of every page, in case the pages become separated. Textbooks, notes and note cards are **not allowed**. Only scientific calculators are allowed. Do your work in the blank spaces and back of pages of this booklet. Show all your work. There are 14 machine-graded problems worth 12 points each and 6 hand-graded problems worth 132 points together for a total of 300 points.

**INSTRUCTIONS FOR MACHINE-GRADED PART (Questions 1-14):**

You MUST use a soft pencil (No. 1 or No. 2) to answer this part. Do not fold or tear the answer sheet, and carefully enter all the requested information according to the instructions you receive. **Do not make any stray marks on the answer sheet.** When you have decided on a correct answer to a given question, circle the answer in this booklet and blacken completely the corresponding circle in the answer sheet. If you erase something, do so completely. Each question has a correct answer. If you give two different answers, the question will be marked wrong. There is no penalty for guessing.

**INSTRUCTIONS FOR THE HAND-GRADED PART (Questions 15-20):**

**SHOW ALL WORK.** You must show all steps in your solutions and make your reasoning clear to earn credit.

**NOTICE REGARDING THE MACHINE GRADED PORTION OF THIS EXAM:**

Either the student or the School of Mathematics may for any reason request a regrade of the machine graded part. All regrades will be based on responses in the test booklet, and not on the machine graded response sheet. Any problem for which the answer is not indicated in the test booklet, or which has no relevant accompanying calculations will be marked wrong on the regrade. *Therefore work and answers must be clearly shown on the test booklet.*

**AFTER YOU FINISH BOTH PARTS OF THE EXAM:** Place the answer sheet between two pages of this booklet (make a sandwich), with the side marked "GENERAL PURPOSE ANSWER SHEET" facing **DOWN**. Have your ID card in your hand when turning in your exam.

Total \_\_\_\_\_

Letter Grade \_\_\_\_\_

Problem	
15	
16	
17	
18	
19	
20	
Subtotal	
1-14	

1. If  $f(x) = x^3 + e^x + \ln x + 5$ , then  $f'(x) =$

A.  $\frac{x^4}{4} + e^x + \ln \frac{x^2}{2} + 5x$

B.  $3x^2 + e^x + \frac{1}{x}$

C.  $3x + xe^{x-1} + \frac{1}{x} + 5$

D.  $x^2 + e^{x-1} + \ln x + 5$

E. None of the above

2. What is the domain of the function  $g(x) = \frac{x-5}{\sqrt{x+2}}$ ?

A. All real numbers

B. All real numbers  $x$  except  $x = -2$

C. All real numbers  $x$  except  $x = -2, x = 5$

D. All real numbers  $x$  for which  $x > -2$

E. All real numbers  $x$  for which  $x \leq -2$

3.  $\lim_{x \rightarrow \infty} \frac{-3x^4 - 5}{10x^2 + 2x + 3} =$

A.  $-\infty$

B.  $\infty$

C. 0

D.  $-\frac{3}{10}$

E.  $-\frac{5}{3}$

4. If  $f(x) = 3x^2 + \sqrt{x}$ , then  $\lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} =$

- A. 0
- B. 6
- C.  $\frac{13}{2}$
- D.  $\frac{\sqrt{5}}{2}$
- E. Does not exist

5.  $\lim_{x \rightarrow 4} \frac{x^2 - 4x}{\sqrt{x} - 2} =$

- A. 0
- B. 2
- C. 4
- D. 16
- E. Does not exist

6. The equation  $(2x + y)^3 = x^2 + 7$  describes a curve in the  $xy$ -plane. What is the slope of the tangent line to this curve at the point where  $x = 1$ ,  $y = 0$ ?

- A. -2
- B. 1
- C.  $\frac{1}{3}$
- D.  $-\frac{5}{3}$
- E.  $-\frac{11}{6}$

7. Let  $f(x) = \begin{cases} 3x + 2 & \text{if } x > 1 \\ 2x^2 - 1 & \text{if } x \leq 1 \end{cases}$  Which of the following is true?

A.  $\lim_{x \rightarrow 1} f(x) = 5$ , and  $f(x)$  is continuous at  $x = 1$ .

B.  $\lim_{x \rightarrow 1} f(x)$  does not exist, and  $f(x)$  is discontinuous at  $x = 1$

C.  $\lim_{x \rightarrow 2} f(x) = 7$ , and  $f(x)$  is continuous at  $x = 2$

D.  $\lim_{x \rightarrow 2} f(x) = 8$ , and  $f(x)$  is discontinuous at  $x = 2$

E. None of the above

8. The improper integral  $\int_1^{\infty} \frac{1}{(2x+1)^3} dx$

A. converges and has value  $\frac{1}{36}$

B. converges and has value  $\frac{1}{27}$

C. converges and has value  $\frac{1}{18}$

D. converges and has value  $3 \ln 3$

E. diverges

9. The average value of the function  $y = \sqrt{x}(x+1)$  over the interval  $0 \leq x \leq 1$  is

A.  $\sqrt{2}$

B.  $\frac{\sqrt{2}}{2}$

C. 1

D.  $\frac{8}{15}$

E.  $\frac{16}{15}$

10. The area of the region enclosed by the curves  $y = x^2 - 4$  and  $y = -2 - x^2$  is

A.  $\frac{32}{3}$

B.  $\frac{16}{3}$

C.  $\frac{8}{3}$

D. 16

E. 8

11. The derivative of  $f(x) = (x^2 + 1)^3(2x^2 - 1)$  evaluated at  $x = 1$  is

A. 8

B. 44

C. 48

D. 56

E. 96

12. If  $f(x, y) = \frac{2xy}{x + 3y}$ , the partial derivative  $f_y$  evaluated at  $x = 1, y = 1$  is

A.  $\frac{1}{2}$

B.  $\frac{1}{8}$

C.  $\frac{2}{3}$

D.  $\frac{3}{4}$

E.  $\frac{7}{16}$

13. Consider the function  $f(x, y) = x^2 + 2y^2 - 4xy + 6x$ . Which of the following is true?

- A.  $f(x, y)$  has a saddle point at  $(0, 0)$ .
- B.  $f(x, y)$  has a relative maximum at  $(-2, -4)$ .
- C.  $f(x, y)$  has a saddle point at  $(3, 3)$ .
- D.  $f(x, y)$  has a relative maximum at  $(-2, -4)$ .
- E.  $f(x, y)$  has a relative minimum at  $(3, 3)$ .

14. A certain bacteria culture grows exponentially. Suppose there are 1000 bacteria initially and 4000 bacteria after two hours. What is the number of bacteria after 3 hours?

- A. 6000
- B. 8000
- C. 10000
- D. 12000
- E. 16000

15. (20 pts) Find the absolute maximum and absolute minimum of the function  $f(x) = \frac{2x}{x^2 + 3}$  on the interval  $[-1, 3]$ .

16. (25 pts) Consider the function  $f(x) = x^3 - 12x + 3$ .

(a) Find the intervals of increase and decrease.

(b) Find all inflection points and determine the intervals of concavity.

(c) Find the relative extrema of  $f(x)$ .

(d) Sketch a graph of  $f(x)$  showing all the information obtained above.

17. (20 pts) Find the particular solution of the differential equation

$$\frac{dy}{dx} = (x - 3)e^{-2y}$$

satisfying  $y = \ln 2$  when  $x = 0$ . Please express  $y$  as a function of  $x$  explicitly.

18. (16 pts) Use logarithmic differentiation to find the derivative of

$$f(x) = 2xe^{-x}(3x + 4)^5.$$

(b) (16 pts) Use integration by parts to find the integral

$$\int \frac{x \ln x}{2} dx.$$

19. (20 pts) A box with an open top and a square base has a surface area 192 square centimeters. What is the largest possible volume of the box? Please justify your answer.

20. (15 pts) Use the formula

$$\int \frac{1}{\sqrt{a^2 + u^2}} du = \ln \left| u + \sqrt{a^2 + u^2} \right| + C$$

to find the definite integral

$$\int_0^1 \frac{6}{\sqrt{3 + 9x^2}} dx.$$