

Math 1142
Spring 2005
FINAL EXAM

Name (Print) _____

Signature _____

Recitation Instructor _____ Section _____ I.D.# _____

READ AND FOLLOW THESE INSTRUCTIONS

This booklet contains 11 pages, including this cover page. Check to see if any are missing. PRINT on the upper right-hand corner all the requested information, and sign your name. Put your initials on the top of every page, in case the pages become separated.

~~Calculator is not permitted. Only numeric calculators are allowed.~~ Do your work in the blank spaces and back of pages of this booklet. Show all your work.

There are 12 multiple choice problems at 12 points each and 6 hand-graded questions with points per problem indicated, for a total of 300 points.

INSTRUCTIONS FOR MACHINE-GRADED PART (Questions 1-12):

You MUST use a soft pencil (No. 1 or No 2) to answer this part. Do not fold or tear the answer sheet, and carefully enter all the requested information according to the instructions you receive. **DO NOT MAKE ANY STRAY MARKS ON THE ANSWER SHEET.** When you have decided on a correct answer to a given question, circle the answer in this booklet and blacken completely the corresponding circle in the answer sheet. If you erase something, do so completely. Each question has a correct answer. If you give two different answers, the question will be marked wrong. There is no penalty for guessing, but if you don't answer a question, skip the corresponding line in the answer sheet. Go on to the next question.

INSTRUCTIONS FOR THE HAND-GRADED PART (Questions 13-18):

SHOW ALL WORK. Unsupported answers will receive little credit.

Notice regarding the machine graded sections of this exam: Either the student or the School of Mathematics may for any reason request a regrade of the machine graded part. All regrades will be based on responses in the test booklet, and not on the machine graded response sheet. Any problem for which the answer is not indicated in the test booklet, or which has no relevant accompanying calculations will be marked wrong on the regrade. *Therefore work and answers must be clearly shown on the test booklet.*

AFTER YOU FINISH BOTH PARTS OF THE EXAM; Place the answer sheet between two pages of this booklet (make a sandwich), with the side marked "GENERAL PURPOSE ANSWER SHEET" facing **DOWN**. Have your ID card in your hand when turning in your exam.

Multiple choice part _____ Hand-graded part _____

Total _____

Letter Grade _____

Problem	
13	
14	
15	
16	
17	
18	
Subtotal	

1. $\lim_{x \rightarrow \infty} \frac{x+1}{2x^2+4} =$

A. 0

B. 1

C. $\frac{1}{2}$

D. $\frac{1}{4}$

E. ∞

2. $f(x) = \frac{2x+3}{x^2+1}$. What is $f'(1)$?

A. $\frac{5}{2}$

B. 2

C. 1

D. $-\frac{3}{2}$

E. $-\frac{5}{2}$

3. If $y = \sqrt{u}$, and $u = (2x^2 + 5)^3$, then $\frac{dy}{dx} =$

A. $\frac{3}{2}(2x^2 + 5)$

B. $\frac{3}{2}\sqrt{2x^2 + 5}$

C. $6x\sqrt{2x^2 + 5}$

D. $\sqrt{3}(2x^2 + 5)$

E. $(2x^2 + 5)^{\frac{3}{2}}$

4. What is the slope of the tangent line to the curve $y = 2\left(\frac{1}{x} + 1\right)(x^2 - 5x + 3)$ at the point where $x = 1$?

A. 6

B. 2

C. -2

D. -4

E. -10

5. $f(x) = \begin{cases} (x^2 - 9)/(x - 3) & \text{if } x < 3 \\ x - 3 & \text{if } x \geq 3 \end{cases}$ Find $\lim_{x \rightarrow 3^-} f(x)$.

A. 6

B. 3

C. 0

D. -3

E. Does not exist

6. Evaluate the definite integral $\int_1^2 x(x^2 - 1)^4 dx$.

A. $\frac{1}{10}$

B. $\frac{81}{10}$

C. 81

D. 162

E. $\frac{243}{10}$

7. The average value of $f(x) = x(x - 2)$ over $0 \leq x \leq 2$ is

A. -4

B. $-\frac{4}{3}$

C. $-\frac{2}{3}$

D. 0

E. 1

8. Let $f(x) = e^{x^3 - 3x}$. Which of the following is true?

A. $f(x)$ is increasing everywhere.

B. $f(x)$ is decreasing everywhere.

C. $f(x)$ is increasing on $(-\infty, -1)$ and $(1, \infty)$, and decreasing on $(-1, 1)$.

D. $f(x)$ is increasing on $(-1, 1)$, and decreasing on $(-\infty, -1)$ and $(1, \infty)$.

E. $f(x)$ is increasing on $(0, 3)$, and decreasing on $(-\infty, 0)$ and $(3, \infty)$.

9. Let $f(x) = x^4 - 4x^3 + 10$. Which of the following is true?

A. $f(x)$ is concave upward on $(3, \infty)$, and concave downward on $(-\infty, 3)$.

B. $f(x)$ is concave upward on $(-\infty, 0)$ and $(3, \infty)$, and concave downward on $(0, 3)$.

C. $f(x)$ is concave upward on $(0, 2)$, concave downward on $(-\infty, 0)$ and $(2, \infty)$.

D. $f(x)$ is concave upward on $(-\infty, 0)$ and $(2, \infty)$, and concave downward on $(0, 2)$.

E. $f(x)$ is concave upward on $(0, 3)$, and concave downward on $(-\infty, 0)$ and $(3, \infty)$.

10. If $f(x, y) = \ln(x^2 + y^2 + 1)$, the partial derivative f_x evaluated at $x = 1$, $y = 0$ is

- A. 0
- B. 1
- C. 2
- D. $\ln 2$
- E. $\ln 3$

11. Consider the function $f(x, y) = x^2 + 2y^2 - xy + 6$. Which of the following is true?

- A. $f(x, y)$ has a saddle point at $(1, 1)$.
- B. $f(x, y)$ has a relative maximum at $(0, 6)$.
- C. $f(x, y)$ has a saddle point at $(0, 6)$.
- D. $f(x, y)$ has a relative maximum at $(0, 0)$.
- E. $f(x, y)$ has a relative minimum at $(0, 0)$.

12. The equation $x^2 - xy + 2y^2 = 2$ defines y implicitly as a function of x . Find $\frac{dy}{dx}$ by implicit differentiation.

- A. $\frac{dy}{dx} = \frac{y-2x}{4y-x}$
- B. $\frac{dy}{dx} = \frac{y+2x}{2y-x}$
- C. $\frac{dy}{dx} = 2x - y$
- D. $\frac{dy}{dx} = \frac{2x}{1+4y}$
- E. $\frac{dy}{dx} = \frac{2x}{1-4y}$

13. (25 pts) Find the area of the region enclosed by the curves $y = x^2 - 2x$ and $y = -x^2$.

14. (25 pts) Use Simpson's rule with $n = 4$ to approximate the definite integral

$$\int_{-2}^2 \frac{4}{x^2 + 4} dx.$$

(Simpson's Rule: $\int_a^b f(x) dx \approx \frac{\Delta x}{3}[f(x_1) + 4f(x_2) + 2f(x_3) + 4f(x_4) + \cdots + 2f(x_{n-1}) + 4f(x_n) + f(x_{n+1})]$, where $\Delta x = \frac{b-a}{n}$ and $x_i = a + (i-1)\Delta x$.)

15. (25 pts) Find the particular solution of the differential equation satisfying

$$\frac{dy}{dx} = xe^{-2y}, \quad y = 0 \text{ when } x = 0.$$

16. (25 pts) Find the absolute maximum and absolute minimum value of $f(x) = 2x + \frac{8}{x}$ for $1 \leq x \leq 3$.

17. (25 pts) Find two numbers x and y so that $x + 2y = 36$ and xy is as large as possible.

18. (a) (15 pts) Use the formula

$$\int \sqrt{a^2 + u^2} du = \frac{u}{2} \sqrt{a^2 + u^2} + \frac{a^2}{2} \ln \left| u + \sqrt{a^2 + u^2} \right| + C$$

to find

$$\int \sqrt{100 + 25x^2} dx.$$

(b) (16 pts) Use integration by parts to find the integral

$$\int x \ln x^3 dx.$$