

Limits

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Limit Rules

Let $f(x)$ and $g(x)$ be any two functions and a be a *real number*. Suppose that c is a *constant* and the limits $\lim_{x \rightarrow a} f(x)$ and $\lim_{x \rightarrow a} g(x)$ exist. Then

1. $\lim_{x \rightarrow a} c = c$

2. $\lim_{x \rightarrow a} x = a$

3. $\lim_{x \rightarrow a} [c \cdot f(x)] = c \cdot \lim_{x \rightarrow a} f(x)$

4. $\lim_{x \rightarrow a} [f(x) + g(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$

5. $\lim_{x \rightarrow a} [f(x) - g(x)] = \lim_{x \rightarrow a} f(x) - \lim_{x \rightarrow a} g(x)$

6. $\lim_{x \rightarrow a} [f(x) \cdot g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$

7. $\lim_{x \rightarrow a} [f(x)/g(x)] = \lim_{x \rightarrow a} f(x) / \lim_{x \rightarrow a} g(x)$ provided $\lim_{x \rightarrow a} g(x) \neq 0$

8. Non-theorem for limits of fractions:

Suppose that $\lim_{x \rightarrow a} f(x) = L \neq 0$ and $\lim_{x \rightarrow a} g(x) = 0$.

Then $\lim_{x \rightarrow a} [f(x)/g(x)]$ is not a real number. That is, it is either $+\infty$ or $-\infty$ or it does not exist.

9. $\lim_{x \rightarrow 0} \frac{1}{x}$ does not exist ; $\lim_{x \rightarrow 0} \frac{1}{x^2} = \infty$; $\lim_{x \rightarrow 0} \frac{x}{x} = 1$; $\lim_{x \rightarrow 0} \frac{x}{x^2}$ does not exist ; $\lim_{x \rightarrow 0} \frac{x}{x^3} = \infty$;

10. $\lim_{x \rightarrow a} x^n = a^n$ and $\lim_{x \rightarrow a} [f(x)]^n = [\lim_{x \rightarrow a} f(x)]^n$

11. $\lim_{x \rightarrow a} x^{1/n} = a^{1/n}$ and $\lim_{x \rightarrow a} [f(x)]^{1/n} = [\lim_{x \rightarrow a} f(x)]^{1/n}$ provided the n -th root is defined

12. $\lim_{x \rightarrow -1} \sqrt{x}$ does not exist

13. Non-theorem for limits at infinity of rational functions:

Suppose that $p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x^1 + a_0$ and $q(x) = b_m x^m + b_{m-1} x^{m-1} + \dots + b_1 x^1 + b_0$.

Then $\lim_{x \rightarrow \infty} \frac{p(x)}{q(x)} = \lim_{x \rightarrow \infty} \frac{a_n x^n}{b_m x^m} = \frac{a_n}{b_m} \lim_{x \rightarrow \infty} x^{n-m}$

Limit Examples

1. $\lim_{x \rightarrow 0} \frac{x^2 - 9}{x + 3} = -3$

2. $\lim_{x \rightarrow 6} \frac{x^2 - 9}{x + 3} = 3$

3. $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x + 3} = 0$

4. $\lim_{x \rightarrow -3} \frac{x^2 - 9}{x + 3} = 6$

5. $\lim_{x \rightarrow 3} \frac{x^2 + 9}{x + 3} = 3$

6. $\lim_{x \rightarrow -3} \frac{x^2 + 9}{x + 3}$ does not exist

7. $\lim_{x \rightarrow 1} \sqrt{x^2 - 2}$ does not exist